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SERIERS REPRESENTATION AND ASYMPTOTICS FOR TITCHMARSH-WEYL m-FUNCTIONS

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Abstract. Sturm-Liouville and Dirac operators are considered on $a \leq x < \infty$, and it is assumed that the coefficients are integrable on $[a, \infty)$. The Titchmarsh-Weyl m-function is given as the ratio of two series which converge on $\operatorname{Im}\lambda \geq 0$ for $|\lambda|$ sufficiently large. For coefficients where N-th derivatives exist and are integrable on $[a, \infty)$, an asymptotic expansion to order λ^{-N} is derived which is valid on $\operatorname{Im}\lambda \geq 0$ as $\lambda \to \infty$. Proofs are given in detail only for the Dirac system. These results contrast with previous work in that the representations are valid for real as well as complex λ.

1. Introduction. A recent paper by Bennewitz [1] develops a convergent series expression for the Titchmarsh-Weyl m-function for the equation

$$-u'' + qu = \lambda u, \qquad 0 \leq x < \infty. \tag{1.1}$$

The function $q \in \mathcal{L}^1(0, \infty)$, and it is proved that the series expression for $m(\lambda)$ converges outside any parabolic region containing the positive real half-axis if $|\lambda|$ is sufficiently large. Further results are obtained, and in particular, if $(1 + t)q(t) \in \mathcal{L}^1(0, \infty)$, the series expression converges for all λ with $|\lambda|$ sufficiently large. For q only locally integrable in $[0, \infty)$, Bennewitz combines his theorem with an estimate of Kaper and Kwong [7] on the diameter of the Weyl disk to establish an asymptotic series for $m(\lambda)$ in certain regions of $\operatorname{Im}\lambda > 0$ which are larger than sectors. This requires only local integrability on q.

While the series of [1] is essentially that considered by Harris [4] and Kaper and Kwong [7-8], it is significant that under certain conditions the series converges for real λ with $|\lambda|$ sufficiently large. This enables one, for example, to find, for certain q, series and asymptotic expressions for the derivative of the spectral function ρ.

The approach of [1] is by the Riccati equation for (1.1). Motivated by [1], we apply here a modified version of the Riccati equation methods of [1] to the Dirac system,

$$y' = \begin{pmatrix} p(x) & \lambda + c + v_1(x) \\ -\lambda + c - v_2(x) & -p(x) \end{pmatrix} y, \quad y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \ a \leq x < \infty, \tag{1.2}$$

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