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HIERARCHICAL MATRIX LYAPUNOV FUNCTION

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Abstract. For the dynamical system

$$\dot{x} = f(t, x, P)$$

describing perturbed motion under structural perturbations, a hierarchical matrix Lyapunov function is proposed which corresponds to a definite level of decomposition of the system.

A vector Lyapunov function is shown to correspond to the first level of decomposition, a matrix Lyapunov function is naturally introduced on the second level and new constructions of auxiliary functions appear on the other levels.

1. Introduction. The hierarchical organization of complex systems in animate and inanimate nature was mentioned by many authors (see Simon [11], Bronowsky [1], Levins [6], Moiseyev [10]). It is essential that the complex stable systems consist of stable subsystems and are apparently subjected to the energy minimal dissipation principle [10] during their evolution.

Our purpose is to develop a direct Lyapunov method which would take account of the hierarchical structure of a complex system and where every level of hierarchy and interconnections has a corresponding "component" of a matrix Lyapunov function. As the system is integrated into a whole, a single hierarchical Lyapunov function is constructed.

The notion of hierarchical Lyapunov function was introduced (see [4]) for a deeper decomposition of complex system. At the same time, when integrating "elementary" subsystems, it is interesting to consider the hierarchical function in order to make clear the effects of, for example, interconnected elementary couples of subsystems on the stability properties of the overall system.

The notion of a hierarchical matrix Lyapunov function is a generalization of the notion of a matrix Lyapunov function (see Martynyuk, Gutowsky [9], Martynyuk [7], Djordjević [2], Grujić, Martynyuk, Ribbens-Pavella [3]). A space matrix-function appears on the higher levels of integration, for example, of three subsystems.

A simple example of a linear system proves that the second level decomposition, together with the hierarchical matrix-function, forms a more flexible technique as compared with the method based on the vector Lyapunov function.

2. Hierarchical decomposition. We consider a dynamical system, the behavior of which is described by (see [3], p. 158)

$$\sigma : \frac{dx}{dt} = f(t, x, P), \tag{2.1}$$

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