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GENERAL FUNCTIONAL DIFFERENTIAL SYSTEMS WITH ASYMPTOTICALLY CONSTANT SOLUTIONS

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Recently the asymptotic properties of solutions of systems of functional differential equations have begun to be studied. (See, e.g., [1]–[10].) Here we give sufficient conditions for rather general functional differential systems to have solutions that approach given constant vectors as $t \to \infty$. This question has been thoroughly investigated for ordinary differential equations, and it seems clear that the methods applied to them can be adapted to functional equations. For example, in [10] the author and T. Kusano obtained sufficient conditions for a functional differential system of the form

$$x_i'(t) = f_i(t, x_1(g_{i1}(t)), \ldots, x_n(g_{in}(t))), \quad 1 \leq i \leq n, \tag{1}$$

to have solutions which approach constant vectors as $t \to \infty$, given that $f_i : [a, \infty) \times \mathbf{R}^n \to \mathbf{R}$ and $g_{ij} : [a, \infty) \to \mathbf{R}$, $1 \leq i,\ j \leq n$, are continuous and that

$$|f_i(t, \xi_1, \ldots, \xi_n)| \leq w_i(t, |\xi_1|, \ldots, |\xi_n|), \quad 1 \leq i \leq n, \tag{2}$$

where $w_1, \ldots, w_n$ satisfy certain monotonicity and integrability conditions.

Assumptions of this kind are standard in connection with systems

$$x_i'(t) = f_i(t, x_1(t), \ldots, x_n(t)), \quad 1 \leq i \leq n,$$

of ordinary differential equations. It is interesting that they lead to results for the system (1), in which no assumptions other than continuity are imposed on the deviating arguments $\{g_{ij}\}$. Nevertheless, although (1) is a considerably more general system than (2), it is not very general in the context of functional differential equations, which may take a great variety of forms. For example, one may wish to consider a functional system

$$X'(t) = F(t; X), \tag{3}$$

or, in component form,

$$x_i'(t) = f_i(t; X), \quad 1 \leq i \leq n$$

(here $X = [x_1, \ldots, x_n]$), in which each $f_i(t; X)$ depends on X evaluated at several (perhaps infinitely many) values of the independent variable t. Therefore, it would seem to be useful to replace assumptions like those stated above by conditions which are easy to check for specific systems, but are not strictly limited in their applicability to systems of a given special form. This is our objective here.

We begin with the following definition from [10].

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