Differential and Integral Equations, Volume 2, Number 4, October 1989, pp. 397–404.

OPTIMAL INTERVALS FOR THIRD ORDER LIPSCHITZ EQUATIONS

PAUL W. ELOE
Department of Mathematics, University of Dayton, Dayton, Ohio 45469 USA

JOHNNY HENDERSON
Department of Algebra, Combinatories & Analysis, Auburn University, Auburn, AL 36849 USA

(Submitted by: Jean Mawhin)

Abstract. For the third order differential equation $y''' = f(t, y, y', y'')$, where

$$|f(t, y_1, y_2, y_3) - f(t, z_1, z_2, z_3)| \leq \sum_{i=1}^{3} k_i |y_i - z_i|$$

on $(a, b) \times \mathbf{R}^3$, subintervals of (a, b) of maximal length are characterized, in terms of the Lipschitz coefficients k_i, $i = 1, 2, 3$, on which certain boundary value problems possess unique solutions. The techniques for determining best interval length involve applications of the Pontryagin Maximum Principle along with uniqueness implies existence arguments.

1. Introduction. In this paper, we are concerned with solutions of boundary value problems for the third order differential equation

$$y''' = f(t, y, y', y''), \tag{1}$$

satisfying

$$y'(t_1) = y_1, \quad y(t_2) = y_2, \quad y'(t_3) = y_3, \quad a < t_1 \leq t_2 \leq t_3 < b, \tag{2}$$

where we assume that f is continuous on the slab $(a, b) \times \mathbf{R}^3$ and satisfies the Lipschitz condition

$$|f(t, y_1, y_2, y_3) - f(t, z_1, z_2, z_3)| \leq \sum_{i=1}^{3} k_i |y_i - z_i| \tag{3}$$

on the slab.

Aftabizadeh, Gupta and Xu [1] recently studied the existence and uniqueness of solutions of (1), (2) under conditions for which Leray-Schauder continuation theory was applicable. Partial motivation for their paper involved a boundary value problem of the form (1), (2) describing the deflection of an equally-loaded beam composed of three parallel layers of different materials.

Received June 16, 1988.
AMS Subject Classifications: 34B10, 34B15, 49A10.