Differential and Integral Equations, Volume 2, Number 4, October 1989, pp. 389–396.

A CORRECT PROBLEM AT A RESONANCE

V. Šeda

Department of Mathematical Analysis, Comenius University at Bratislava
842-15 Bratislava, Mlynská dolina, Czechoslovakia

(Submitted by: Klaus Schmitt)

Abstract. A method based on a surjectivity result in R^n is developed by means of which the correctness of a boundary value problem is proved. The corresponding homogeneous problem has a one dimensional space of solutions. In the method, the Kamke convergence lemma and the Sedziwy lemma on the continuity on boundary values of solutions to a boundary value problem are used.

In this paper, the correctness of the boundary value problem

$$x'' = f(t, x, x') \tag{1}$$

$$x'(a) = A, \quad x(b) - x(t_0) = B, \tag{2}$$

where $a < t_0 < b$, A, B are given real numbers and $f \in C([a,b] \times R^2, R)$, will be shown. Related problems have been studied in [3], [4].

First, we give a sufficient condition for the uniqueness of that boundary value problem.

Lemma 1. *Assume that*

(i) $f(t, \cdot, y)$ *is nondecreasing in* R *for each* $(t, y) \in [a,b] \times R$;

(ii) *for each* $r > 0$, *there is an* $L_r > 0$ *such that*

$$|f(t,x,y) - f(t,x,z)| \le L_r |y - z|$$

for any two points (t,x,y), $(t,x,z) \in [a,b] \times [-r,r] \times [-r,r]$.

Then the following statement holds:

If $x(t)$ *and* $y(t)$ *are two solutions of (1) on* $[a,b]$ *and* $x(t) - y(t) \ge 0$ *in* $[t_1, t_2] \subset [a,b]$, $x'(t_1) - y'(t_1) > 0$ $(x'(t_1) - y'(t_1) = 0)$, *then* $x(t) - y(t) > 0$, $x'(t) - y'(t) > 0$ *in* $(t_1, b]$, $(x'(t) - y'(t) \ge 0$ *in* $[t_1, t_2])$.

Proof: Denote $v(t) = x(t) - y(t)$ for $t \in [a,b]$. Then,

$$v''(t) = [f(t, x(t), x'(t)) - f(t, y(t), x'(t))] + [f(t, y(t), x'(t)) - f(t, y(t), y'(t))] \tag{3}$$

Received April 5, 1988, Revised January 22, 1989.
AMS Subject Classifications: 34B15, 34B10.