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ON THE ZEROS OF SOLUTIONS OF HYPERBOLIC EQUATIONS OF NEUTRAL TYPE

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Abstract. Hyperbolic equations of neutral type are studied and sufficient conditions are given that every solution of certain boundary value problems has a zero in bounded domains. The results are based on the condition for the non-existence of positive solutions of ordinary differential inequalities.

Recently there has been an increasing interest in studying the oscillatory behavior of solutions of partial differential equations of neutral type (see [1–3]). To the author's knowledge, the first attempt in this direction was made by Mishev and Bainov [1] who studied the hyperbolic equation of neutral type.

Let G be a bounded domain in $\mathbb{R}^n$ with smooth boundary ∂G, and let $\Omega = G \times (0, \infty)$. We are concerned with the oscillatory behavior of solutions of the hyperbolic equation of neutral type

$$u_{tt}(x,t) - [\Delta u(x,t) + \alpha \Delta u(x, t-\tau)] + c(x,t,u(x,t),u(x,t-\sigma)) = f(x,t), \tag{1}$$

$(x,t) \in \Omega$, where Δ is the Laplacian in $\mathbb{R}^n$. We consider three kinds of boundary conditions:

$$u = \psi \quad \text{on } \partial G \times (0,\infty); \tag{B$_1$}$$

$$\frac{\partial u}{\partial \nu} = \tilde{\psi} \quad \text{on } \partial G \times (0,\infty); \tag{B$_2$}$$

$$\frac{\partial u}{\partial \nu} + \mu u = 0 \quad \text{on } \partial G \times (0,\infty), \tag{B$_3$}$$

where ψ, $\tilde{\psi}$ are continuous functions on $\partial G \times (0, \infty)$, μ is a nonnegative continuous function on $\partial G \times (0,\infty)$ and ν denotes the unit exterior normal vector to ∂G. In [1], Mishev and Bainov obtained sufficient conditions for the existence of arbitrarily large zeros of solutions of the problem (1), (B$_2$). The purpose of this paper is to

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