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STABILITY OF DYNAMICAL SYSTEMS IN THE PLANE

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Abstract. In this paper stability and asymptotic stability of the real system $x' = A(t)x + h(t,x)$, as well as asymptotic behavior of its solutions are studied. Here, $A(t)$ is a square matrix and $h(t,x)$ is a vector function. The method: the system is recasted to an equation with complex conjugate coordinates and this equation is studied by means of a suitable Lyapunov-like function. Applications to a linear differential equation of the second order are given.

1. Introduction. Consider the real dynamical system in the plane

$$x' = A(t)x + h(t,x), \tag{1}$$

where $A(t) = (a_{jk}(t))$, $j,k = 1,2$ is a square matrix and $h(t,x) = \big(h_1(t,x),\ h_2(t,x)\big)$ is a vector function. We suppose that a_{jk} have continuous first derivatives on $[t_0,\infty)$ and that h is continuous on

$$[t_0,\infty) \times \{[x_1,x_2] \in R^2 : x_1^2 + x_2^2 < r \leq \infty\}.$$

In addition, we suppose the uniqueness of any initial value problem for (1).

Our purpose is to present in this paper a new method suitable for the study of stability problems of dynamical systems in the plane. Although one can apply to (1) methods elaborating for the n-dimensional case (see Coppel [2], for example), our method leads in two-dimensional case to new, very effective and easy applicable results. A similar method was used in papers by Ráb [12], [13], [14] to the study of a Riccati differential equation with complex-valued coefficients and generalized by Kalas for more general equations [3]-[10]. The stability of (1) has been also studied by Osička [11] and Radzikowski [15] in the case $h(t,x) \equiv 0$.

The system (1) will be recasted into an equivalent complex form involving both $z = x_1 + ix_2$ and the conjugate variable $\bar{z} = x_1 - ix_2$. The real plane is converted into the complex plane C by assigning the complex number $z = x_1 + ix_2$ to the point $[x_1,x_2]$. In matrix form the transformation from real to conjugate coordinates can be written

$$y = Bx\,, \quad B = \begin{bmatrix} 1 & i \\ 1 & -i \end{bmatrix},$$

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