DYNAMIC THEORY OF QUASILINEAR PARABOLIC EQUATIONS
II. REACTION-DIFFUSION SYSTEMS

Herbert Amann
Mathematisches Institut, Universität Zürich, Rämistrasse 74, CH-8001 Zürich, Switzerland

(Submitted by: M.G. Crandall)

Introduction. Let \( \Omega \) be a bounded smooth domain in \( \mathbb{R}^n \) and consider a second order differential equation of the form

\[
\partial_t u - \partial_j (a_{jk}(\cdot, u) \partial_k u) = f(\cdot, u, \partial u) \quad \text{on} \quad \Omega \times (0, \infty)
\]

acting on \( \mathbb{R}^N \)-valued functions \( u = (u^1, \ldots, u^N) \). (We use the summation convention throughout, \( j \) and \( k \) running from 1 to \( n \), and \( r \) and \( s \) running from 1 to \( N \).) We assume that

\[
a_{jk} \in C^\infty(\overline{\Omega} \times G, \mathcal{L}(\mathbb{R}^N)), \quad 1 \leq j, k \leq n,
\]

where \( G \) is an open subset of \( \mathbb{R}^N \) and \( \mathcal{L}(\mathbb{R}^N) \) is the space of all real \( N \times N \) matrices. We assume also that

\[
f \in C^\infty(\overline{\Omega} \times G \times \mathbb{R}^{nN}, \mathbb{R}^N)
\]

and that \( f \) is 'affine in the gradient', that is,

\[
f(\cdot, \cdot, \eta) = f_0 + \sum_{j=1}^n f_j \eta_j, \quad \eta := (\eta_1, \ldots, \eta_n) \in \mathbb{R}^N \times \cdots \times \mathbb{R}^N,
\]

where \( f_0 : \overline{\Omega} \times G \to \mathbb{R}^N \) and \( f_j : \overline{\Omega} \times G \to \mathcal{L}(\mathbb{R}^N), 1 \leq j \leq n \).

Equation (1) has to be complemented by boundary conditions, which are typically 'Dirichlet boundary conditions',

\[
u = 0 \quad \text{on} \quad \partial \Omega \times (0, \infty),
\]

or 'Neumann type boundary conditions',

\[
a_{jk}(\cdot, u) \nu^j \partial_k u = g(\cdot, u) \quad \text{on} \quad \partial \Omega \times (0, \infty),
\]

where \( \nu := (\nu^1, \ldots, \nu^n) \) is the outer unit normal vector field on \( \partial \Omega \) and

\[
g \in C^\infty(\partial \Omega \times G, \mathbb{R}^N).
\]