

BIFURCATION FOR PERIODIC DIFFERENTIAL EQUATIONS AT RESONANCE

S.R. BERNFELD

Department of Mathematics, University of Texas, Arlington, Texas 76019 USA

L. SALVADORI

Dipartimento di Matematica, Università di Trento, 38050 Povo (Trento), Italy

F. VISENTIN

*Dipartimento di Matematica e Applicazioni, Università di Napoli
Via Mezzocannone 8, 80134 Napoli, Italy*

Abstract. This paper is concerned with bifurcation and stability problems for a one parameter family of T -periodic differential systems in \mathbb{R}^2 having the origin as an equilibrium point. Under some resonance condition we analyze the existence and stability of T -periodic solutions near the equilibrium point.

1. Introduction. This work is the third of a series of papers dealing with bifurcation and stability for a one parameter family of periodic differential systems in \mathbb{R}^2 . We strengthen and extend some of the results appearing in [1] and [2] as follows. Consider the one parameter system of ordinary differential equations in \mathbb{R}^2

$$\begin{aligned}\dot{x} &= \alpha(\mu)x - \beta(\mu)y + X(t, x, y, \mu) \\ \dot{y} &= \alpha(\mu)y + \beta(\mu)x + Y(t, x, y, \mu),\end{aligned}\tag{S}_\mu$$

where μ is a parameter, X and Y are of order ≥ 2 in (x, y) as $(x, y) \rightarrow 0$ and $X(t+T, x, y, \mu) \equiv X(t, x, y, \mu)$, $Y(t+T, x, y, \mu) \equiv Y(t, x, y, \mu)$ for some fixed $T > 0$. Moreover X, Y, α, β are sufficiently smooth in all their variables, $\alpha(0) = 0$, $\alpha'(0) > 0$ and $\beta(0) > 0$. Assume that system $(S)_0$ is autonomous, and that the origin of $(S)_0$ is asymptotically stable, this property being recognizable by the terms up to some order h in the MacLaurin expansion in x, y of $X(t, x, y, 0)$, $Y(t, x, y, 0)$ (h -asymptotically stable). It was proved in [2] that in a $\mathbb{R} \times \mathbb{R}^2$ neighborhood H of the set $M_0 = \{(t, x, y) \in \mathbb{R} \times \mathbb{R}^2 : t \in \mathbb{R}, x = 0, y = 0\}$ and for each $\mu > 0$ and small the largest invariant set M_μ of $(S)_\mu$ contained in $H - M_0$, whose sections $M_\mu(t)$ with the planes $t = \text{constant}$ are compact, is nonempty, asymptotically stable, and the mapping $t \rightarrow M_\mu(t)$ is T -periodic.

Received January 30, 1989.

Partially supported by Italian Ministry of Education and by Italian C.N.R. (National Council of Research).

AMS Subject Classifications: 34D20, 58F14.