Differential and Integral Equations, Volume 3, Number 2, March 1990, pp. 397–400.

ON NONSMOOTH SOLUTIONS OF ABEL'S INTEGRAL EQUATION

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(Submitted by: Klaus Schmitt)

Consider Abel's integral equation

$$Au(t) = \int_0^t u(s)(t-s)^{-\alpha}\,dx = f(t), \quad 0 < t < 1, \; 0 < \alpha < 1 \tag{1}$$

where f is given in $L_2 = L_2(0,1)$. The importance of this equation is well-known (see [1]). It is also known that solving (1) is, in general, an ill-posed problem, e.g., for $A : L_2 \to L_2$ or for $A : C[0,1] \to C[0,1]$. In [3], the authors gave a regularized solution $u_\beta(t)$ to (1) with the following error estimate

$$|u - u_\beta| \le c\epsilon^{1/2}, \qquad |\cdot| = L_2 - \text{norm}, \tag{2}$$

under the assumption that u is in $H^1 = H^1(0,1)$ and that

$$|Au - f| < \epsilon. \tag{3}$$

In [2], Gorenflow did away with the smoothness condition on the solution u of the equation $Au = f$, which he assumed to exist as a function of bounded variation of a special kind, viz the sum of a Lipschitz-continuous function and a (finite or denumerably infinite) linear combination of Heaviside functions. The solution is constructed by the backward Euler method, with an error estimate which tends to zero as $h^{1-\alpha}$ for h(the mesh size)$\to 0$.

In this note, we shall consider the case of an arbitrary function of bounded variation. In fact, we shall produce a regularized solution u_β of (1) with an error estimate similar to (2) above. It is a pleasure to recall the illuminating conversations on Abel integral equations the second named author (D.D.A.) had with Professor Gorenflo at the ICOMIDC Symposium on Mathematics of Computation.

For our regularization of (1), we note first that $A : L_1 \to L_1 (L_1 = L_1(0,1))$ is of norm $\le (1-\alpha)^{-1}$. Define the operators K and B on L_1 by

$$Ku(t) = \int_0^t u(s)\,ds, \quad Bu(t) = \int_0^t u(s)(t-s)^{-1+\alpha}\,ds. \tag{4}$$

Received October 24, 1988.

†On leave from HoChiMinh City University.

AMS Subject Classifications: 47G05.