Differential and Integral Equations, Volume 3, Number 2, March 1990, pp. 335–351.

CONVERGENCE OF THE LINE METHOD APPROXIMATION FOR A PARABOLIC FREE BOUNDARY PROBLEM

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(Submitted by: K. Schmitt)

Abstract. In this paper, an approximate solution to a diffusion process with free boundary is obtained by applying Rothe's method (method of lines). A sequence of approximate problems is obtained by discretizing the partial differential equations with respect to the time variable. These approximate problems are shown to be uniquely solvable and the solutions are shown to converge to the unique solution of the diffusion process. The existence and convergence of the approximate solutions and the free boundary are obtained using techniques from the theory of differential inequalities.

1. Introduction. In this paper, we show the existence, uniqueness and convergence of line method approximations to the solution of an implicit one-dimensional parabolic free boundary problem with a nonlinear source term

$$\begin{cases} u_{xx} - u_t = f(t, x, u, u_x), \\ au(t,0) - bu_x(t,0) = \alpha(t), \quad u(0,x) = 0 \\ u(t, s(t)) = u_x(t, s(t)) = 0, \quad s(0) = 0. \end{cases} \tag{1}$$

Interest in free boundary problems for the heat equation has been high during the last two decades, resulting in a substantial body of literature. An introduction to these problems as well as extensive bibliographies can be found in the monographs by L.I. Rubinšteĭn [12] and J.R. Cannon [1] (the latter contains references to more recent results) and in the paper by A. Fasano and M. Primicerio [3]. In the majority of these applications, the evolution of the free boundary is determined by a condition which relates the time derivative of the free boundary explicitly in terms of the flux u_x. For example, in the classical Stefan problem, this condition has the form $u_x(t, s(t)) = -\lambda \dot{s}(t)$, where $\dot{s}$ denotes the derivative of s with respect to t. Problem (1) belongs to another class of applications, which has received somewhat less attention, in which the free boundary appears only implicitly. Implicit free

Received November 6, 1988.
AMS Subject Classifications: 35R35, 65N40, 35R45, 35K05, 35K60, 34B15, 34A40.