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A THEOREM OF THE KREIN-RUTMAN TYPE FOR AN INTEGRO-DIFFERENTIAL OPERATOR

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Abstract. It is proved a theorem of the Krein-Rutman type for the problem $-\Delta u + Bu = \lambda mu$ in Ω, $u = 0$ on $\partial\Omega$, where $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain, $B = \delta(-\Delta + \gamma)^{-1}$ under Dirichlet boundary conditions, (δ, γ) belongs to some unbounded region of $(\mathbb{R}^+ - \{0\}) \times (\mathbb{R}^+ - \{0\})$, λ is a real parameter and m is a continuous indefinite weight function assuming a positive value at some point $x_0 \in \Omega$.

1. Introduction. During recent years, many authors have studied reaction-diffusion systems derived from several applications, such as mathematical biology, chemical reactions and combustion theory, among other physical phenomena. See, for example [4], [6] and [9].

In this work, we study the elliptic system

$$\begin{aligned} -\Delta u = \lambda f(x,u) - v \quad \text{in } \Omega, \quad -\Delta v = \delta u - \gamma v \quad \text{in } \Omega, \\ u = v = 0 \quad \text{on } \partial\Omega, \quad \lambda \geq 0 \end{aligned} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain, $f : \bar{\Omega} \times \mathbb{R} \to \mathbb{R}$ is a given nonlinearity and δ, $\gamma \in \mathbb{R}^+ - \{0\}$. The solutions (u, v) of this problem represent steady-state solutions of a reaction-diffusion system derived from mathematical biology. Problem (1) is equivalent to the integro-differential problem

$$\begin{aligned} -\Delta u + \delta(-\Delta + \gamma)^{-1} u = \lambda f(x,u) \quad & \text{in } \Omega, \\ u = 0 \quad & \text{on } \partial\Omega, \ \lambda \geq 0 \end{aligned} \tag{2}$$

where $v = \delta(-\Delta + \gamma)^{-1} u$ and $\delta(-\Delta + \gamma)^{-1}$ is considered under Dirichlet boundary conditions.

To study some questions related to problem (2), such as bifurcation and stability, among other things, very often we need to know some information about eigenvalues and eigenfunctions of the linear eigenvalue problem

$$-\Delta u + \delta(-\Delta + \gamma)^{-1} u = \lambda mu \quad \text{in} \quad \Omega, \quad u = 0 \quad \text{on} \quad \partial\Omega \tag{3}$$

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