Differential and Integral Equations, Volume 3, Number 2, March 1990, pp. 237–252.

AN IDENTIFICATION PROBLEM IN THE THEORY OF HEAT CONDUCTION

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Abstract. We consider the nonlinear one dimensional heat equation in $Q_T = [0,T] \times [0,L]$, where both the temperature u and the conductivity coefficient a are unknown. The temperature and its spatial derivative $D_x u$ are known on the parabolic boundary of Q_T, and the nonlinear function a is known in a certain temperature interval. The inverse problem is reduced to a quasilinear parabolic initial boundary value problem, which is solved by a linearization procedure.

0. Introduction and notation. This paper deals with identifying (in particular, proving the existence of) the nonlinear function a in the parabolic initial boundary value problem

$$D_t u(t,x) = D_x\big(a(u(t,x)) \cdot D_x u(t,x)\big) + f(t,x), \quad (t,x) \in Q_T = [0,T] \times [0,L] \tag{0.1}$$

$$u(t,0) = g_1(t), \quad 0 \le t \le T \tag{0.2}$$

$$u(0,x) = g_2(x), \quad 0 \le x \le L \tag{0.3}$$

$$u(t,L) = g_3(t), \quad 0 \le t \le T. \tag{0.4}$$

It is well-known that for every prescribed (and smooth) positive function a, problem (0.1)–(0.4) admits a unique local solution, provided the data f, g_1, g_2, g_3 are sufficiently regular and satisfy compatibility conditions.

On the contrary, in our case the function a is unknown, so that additional information is needed to determine the pair (u,a). We consider here the following additional conditions:

$$D_x u(t,0) = g_4(t), \quad 0 \le t \le T \tag{0.5}$$

$$D_x u(t,L) = g_5(t), \quad 0 \le t \le T \tag{0.6}$$

$$a(\tau) = a_0(\tau), \quad \tau \in R(g_2) \tag{0.7}$$

Received February 25, 1989.
AMS Subject Classifications: 35K05, 35R30.