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ON THE SOLVABILITY OF A NONLINEAR TWO POINT BVP BETWEEN THE FIRST TWO EIGENVALUES

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Abstract. The solvability of the two-point BVP for the second-order nonlinear differential equation $u''+g(u) = p(t)$ is achieved under new nonresonance conditions with respect to the first two eigenvalues of the associated linear problem.

1. Introduction and statement of the result. In this paper, we provide some new conditions for the solvability of the nonlinear two-point boundary value problem

$$u'' + g(u) = p(x) \tag{1.1}$$

$$u(a) = r_1, \quad u(b) = r_2, \tag{1.2}$$

where $g : \mathbb{R} \to \mathbb{R}$ is a continuous function, $p : [a,b] \to \mathbb{R}$ is Lebesgue integrable and $r_1, r_2 \in \mathbb{R}$. Solutions to (1.1)–(1.2) are considered in the generalised sense and are classical ones whenever $p(\cdot)$ is continuous.

Starting with Dolph [7], a great deal of existence theorems concerning (1.1)–(1.2) has been proved under suitable nonresonance conditions on the nonlinear term g. The usual assumptions require that, asymptotically, the range of $g(s)/s$ does not intersect the spectrum of the differential operator $u \mapsto -u''$, subjected to the homogeneous boundary conditions (see, for instance, [1, 13, 15, 18, 23]). In particular, it is known that (1.1)–(1.2) has at least one solution, for any r_1, r_2 and $p(\cdot)$, provided that

$$\lambda_1 < \liminf_{s\to\pm\infty} \frac{g(s)}{s} \le \limsup_{s\to\pm\infty} \frac{g(s)}{s} < \lambda_2,$$

where λ_1 and λ_2 are the first two eigenvalues of the associated linear problem.

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