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INERTIAL MANIFOLDS AND SACKER'S EQUATION

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1. Introduction. Much progress has been made in the study of dissipative evolution equations because they can be considered as infinite dimensional dynamical systems. For instance, the notion of attractor that existed for finite dimensional systems has been extended and many equations have a global attractor which is compact and connected (see e.g. Temam [20] or Hale [14]).

If $(S(t))_{t\geq 0}$ is the semigroup associated to a dynamical system in a Banach space E, the global attractor is a compact subset of E that attracts all the bounded sets of E,

$$\forall B \subset E,\ B \text{ bounded},\ d(S(t)B, \mathcal{A}) \to 0 \text{ when } t \to +\infty; \tag{1.1}$$

moreover, it is invariant,

$$\forall t \geq 0, \quad S(t)\mathcal{A} = \mathcal{A}. \tag{1.2}$$

Such a set is interesting in order to describe the behavior of the orbits of the system for large time. Indeed, we know that in numerous cases the attractor is finite dimensional and we have very good estimates of its dimension that correspond to the physical situation (see Temam [20]).

But attractors are not well adapted to practical purposes (especially numerical utilizations) for two reasons:
- they can attract the orbits very slowly;
- their geometry can be very complicated (perhaps fractals) and they are not regular objects.

For these reasons, the notion of inertial manifold has been introduced by Foias, Sell and Temam [11–12]. It is defined as follows : let $(S(t))_{t\geq 0}$ be a dynamical system in a Banach space E, a set $\mathcal{M}$ is an inertial manifold for this semigroup if

$$\begin{aligned}
&- \mathcal{M} \text{ is a Lipschitzian manifold,}\\
&- \mathcal{M} \text{ is positively invariant} : \forall t \geq 0, \quad S(t)\mathcal{M} \subset \mathcal{M},\\
&- \mathcal{M} \text{ attracts the orbits of } (S(t))_{t\geq 0} \text{ with an exponential speed,}\\
&- \mathcal{M} \text{ is finite dimensional.}
\end{aligned} \tag{1.3}$$

When an inertial manifold exists, the system is very well approximated by the inertial system that we obtain by restricting $(S(t))_{t\geq 0}$ to $\mathcal{M}$.

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