Differential and Integral Equations, Volume 3, Number 3, May 1990, pp. 401–433.

OSCILLATIONS OF SOLUTIONS OF PERTURBED AUTONOMOUS EQUATIONS WITH AN APPLICATION TO NONLINEAR ELLIPTIC EIGENVALUE PROBLEMS INVOLVING CRITICAL SOBOLEV EXPONENTS

F.V. ATKINSON† AND L.A. PELETIER‡
Argonne National Laboratory, Argonne, IL 60439, USA

Abstract. We discuss radially symmetric, not necessarily positive, solutions of the Dirichlet problem for $\Delta u + \{f_1(u,r) + \lambda f_2(u,r)\} = 0$ in the unit ball $B_N, N > 2$, where f_1 and f_2 are suitably homogeneous functions with f_1 critical and f_2 subcritical. Estimates are obtained for asymptotic relations between $u(0)$ and λ. The method transforms the problem to a study of the zeros of certain solutions of a perturbed autonomous ordinary differential equation.

1. Introduction. As the title indicates, the problem of this paper admits both an ODE and a PDE presentation. While these are very nearly co–extensive, the ODE version has certain advantages, such as freedom from dimensional restrictions. We are concerned in this version with the asymptotics, as $s \to \infty$, of exponentially small solutions of equations of the form

$$w''(s) - w(s) + g(w(s)) + h(w(s), s) = 0, \tag{1.1}$$

where g is a nonlinear function and $h(w,s)$, qua function of s, is exponentially small as $s \to \infty$. Typical examples of recent interest for the PDE application will be such cases as

$$g(w) = w|w|^{p-1}, \quad h(w,s) = w|w|^{q-1}e^{-ms}, \quad 1 \le q < p,\ m > 0. \tag{1.2}$$

However we do not confine our attention to power–type behaviour in w.

We view (1.1) as a perturbation of the autonomous equation

$$w'' - w + g(w) = 0, \tag{1.3}$$

and this in turn as a pertubation of

$$w'' - w = 0 \tag{1.4}$$

Received May 30, 1989.
†Department of Mathematics, University of Toronto, Toronto, M5S 1A1, Canada.
‡Mathematical Institute, Leiden University, PB 9512, 2300 RA Leiden, The Netherlands.
AMS Subject Classifications:34E10, 35J60.