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ON A LOGISTIC EQUATION WITH PIECEWISE CONSTANT ARGUMENTS

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Abstract. Let $[\cdot]$ denote the greatest-integer function and consider the logistic equation with piecewise constant arguments

$$\frac{dN(t)}{dt} = rN(t)\{1 - \sum_{j=0}^{m} a_j N([t-j])\}, \tag{*}$$

where $r \in (0,\infty)$ and $a_0, a_1, \cdots, a_m \in [0,\infty)$ with $\sum_{j=0}^{m} a_j > 0$ and $r+m \neq 1$. We obtained necessary and sufficient conditions for the oscillation of all positive solutions of equation $(*)$ about the positive steady state $N^* = (\sum_{j=0}^{m} a_j)^{-1}$. We also obtained sufficient conditions for the global attractivity of the positive steady state N^*.

1. Introduction.

The delay logistic equation

$$\frac{dx(t)}{dt} = rx(t)\{1 - \frac{x(t-\tau)}{K}\} \tag{1.1}$$

where r, τ and K are positive numbers and several of its generalizations have been considered in the literature on mathematical ecology. Equation (1.1) is one of several models of the dynamics of a single species population capable of demonstrating oscillation of all solutions. Apart from its potential for ecological applications, equation (1.1) has been mathematically interesting and has been investigated by many authors and notably by Wright [9], Jones [5] and Kakutani and Markus [6].

Let $[\cdot]$ denote the greatest-integer function. In this paper, we are concerned with a study of the oscillatory and asymptotic properties of solutions of the equation

$$\frac{dN(t)}{dt} = rN(t)\{1 - \sum_{j=0}^{m} a_j N([t-j])\}, \quad t \geq 0, \tag{1.2}$$

where $r \in (0,\infty)$ and $a_0, a_1, \cdots, a_m \in [0,\infty)$ with $\sum_{j=0}^{m} a_j > 0$ and $r+m \neq 1$.

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