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INTEGRAL AVERAGES AND OSCILLATION OF SECOND ORDER SUBLINEAR DIFFERENTIAL EQUATIONS

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Abstract. The sublinear differential equation $x''(t) + a(t)f[x(t)] = 0$, $t \geq t_0 > 0$ is considered, in which $a \in C([t_0, \infty))$, $f \in C(R)$ with $yf(y) > 0$ for $y \neq 0$ and $\int_{\pm 0}^{\pm 1} [1/f(y)]\, dy < \infty$, and f has a continuous derivative on $R - \{0\}$ with $f'(y) \geq 0$ for all $y \neq 0$. No sign condition is assumed on a. Two new oscillation criteria are obtained. These criteria involve the average behavior of the integral of the coefficient a.

1. Introduction. Many physical systems are modelled by second order nonlinear ordinary differential equations. For example, the so-called Emden-Fowler equation arises in the study of gas dynamics and fluid mechanics. Also, this equation appears in the study of relativistic mechanics, nuclear physics and in the study of chemically reacting systems. The study of the Emden-Fowler equation originates from earlier theories concerning gaseous dynamics in astrophysics around the turn of the century. For a discussion on the Emden-Fowler equation, we refer to Wong [20].

Consider the second order nonlinear ordinary differential equation

$$x''(t) + a(t)f[x(t)] = 0, \tag{E}$$

where a is a continuous function on the interval $[t_0, \infty)$, $t_0 > 0$, and f is a continuous function on the real line $\mathbb{R}$. It will be supposed that f has a continuous derivative on $\mathbb{R} - \{0\}$ and satisfies

$$yf(y) > 0 \quad \text{and} \quad f'(y) \geq 0 \quad \text{for all} \quad y \neq 0.$$

Moreover, we are interested in the case where (E) is strongly sublinear in the sense that

$$\int_{+0} \frac{dy}{f(y)} < \infty \quad \text{and} \quad \int_{-0} \frac{dy}{f(y)} < \infty.$$

Note that no assumption on the sign of the coefficient a is made.

We restrict our attention to solutions of (E) which exist on some ray $[T_0, \infty)$, where $T_0 \geq t_0$ may depend on the particular solution. Such a solution is said to be *oscillatory* if it has arbitrarily large zeros, and otherwise it is said to be *nonoscillatory*. Equation (E) is called oscillatory if all its solutions are oscillatory.

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