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OSCILLATION FOR SELF–ADJOINT SECOND ORDER MATRIX DIFFERENTIAL EQUATIONS

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Abstract. Oscillation theorems are established for the self–adjoint matrix differential equation $(PY')' + QY = 0$. Methods and results are similar to those in references [3] and [4] (the scalar case) and [2] (the matrix case $Y'' + QY = 0$).

1. Introduction. Consider the $n \times n$ matrix differential equation

$$(P(t)Y')' + Q(t)Y = 0 \tag{1}$$

on $[0, +\infty)$, where $P(t)$ and $Q(t)$ are real, continuous, and symmetric, and $P(t) > 0$ ($P(t)$ is positive definite). A solution $Y(t)$ is prepared if

$$Y^*(PY') - (PY')^*Y \equiv 0$$

(* denotes transpose), and (1) is oscillatory on $[0, +\infty)$ provided, for each $a \geq 0$, the determinant of each nontrivial prepared solution has a zero on $[a, +\infty)$.

There are extensions to (1) of the oscillation theory for the scalar equation

$$(p(t)y')' + q(t)y = 0 \tag{2}$$

to (1) (see [1], [2] for references). In particular, a conjecture appeared in [5] that

$$Y'' + Q(t)Y = 0 \tag{3}$$

is oscillatory provided

$$\lambda_1\Big\{\int_0^t Q\Big\} \to +\infty \quad \text{as} \quad t \to \infty$$

(λ_1 is the greatest eigenvalue), a direct analog of the result that

$$y'' + q(t)y = 0 \tag{4}$$

is oscillatory if $\int_0^\infty q = +\infty$ ([6]).

After partial results by several authors (see [2] for references), Byers, Harris and Kwong proved

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