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A FAST DIFFUSION EQUATION WHICH GENERATES A MONOTONE LOCAL SEMIFLOW II: GLOBAL EXISTENCE AND ASYMPTOTIC BEHAVIOR

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Abstract. Global existence and large-time asymptotic behavior of mild solutions to the Cauchy problem for the fast diffusion equation $\partial_t n = d \cdot \partial_x (n^{-1} \cdot \partial_x n)$, $(x, t) \in \mathbb{R} \times \mathbb{R}_+$, with the boundary conditions $\lim_{x \to -\infty} n^{-1} \cdot \partial_x n = c$ and $\lim_{x \to \infty} n = b$ are investigated. Here, $b, c, d \in (0, \infty)$ are given constants. It is proved that, when viewed as an abstract evolution equation in a suitable Sobolev space Y, this problem has a unique mild solution which exists globally in time, is C^∞ in $\mathbb{R} \times (0, \infty)$ and satisfies the boundary conditions for every $t \in \mathbb{R}_+$ whenever $n(x, 0) \in Y$ is given. These solutions form a semigroup of monotone contractions in $\overline{Y}$ = closure of Y in the translation of $L^1(\mathbb{R})$ by the Heaviside step function. Each solution approaches a traveling wave in the $L^1(\mathbb{R})$-metric as $t \longrightarrow \infty$.

1. Introduction. This paper is the second one from a series of two papers studying a fast diffusion equation. The purpose of this paper is to study global (in time) existence and asymptotic behavior of a solution $n(x, t)$ to the Cauchy problem for the following fast diffusion equation on the real line:

$$\partial_t n = d \cdot \partial_x (n^{-1} \cdot \partial_x n) \quad \text{for} \quad -\infty < x < \infty, \quad t > 0; \tag{1.1}$$

$$n(x, 0) = n_0(x) \qquad \text{for} \quad -\infty < x < \infty; \tag{1.2}$$

$$\lim_{x \to -\infty} n^{-1}(x, t) \cdot \partial_x n(x, t) = c \quad \text{for} \quad t > 0; \tag{1.3}$$

$$\lim_{x \to \infty} n(x, t) = b \quad \text{for} \quad t > 0. \tag{1.4}$$

Here, $n : \mathbb{R} \times \mathbb{R}_+ \longrightarrow (0, \infty)$ is the unknown function whose initial value at $t = 0$ is a given function $n_0 : \mathbb{R} \longrightarrow (0, \infty)$, and $b, c, d \in (0, \infty)$ are given constants.

Equation (1.1) arises in a number of nonlinear diffusion problems in mathematical physics and population dynamics, cf. Takáč [12] for references. The boundary condition (1.3) means constant flux at $x = -\infty$, while (1.4) means constant density at $x = \infty$. Equation (1.1) on the bounded interval $(0, 1)$ with Dirichlet boundary conditions $n(0, t) = n(1, t) = b > 0$ $(t > 0)$ and the asymptotic behavior of $n(x, t)$

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