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TIME DEPENDENT m-ACCRETIVE OPERATORS GENERATING DIFFERENTIAL EVOLUTIONS

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Abstract. This paper concerns the existence of the evolution operator for the evolution equation $x'(t) + A(t)x(t) = 0$, where $A(t)$ are m-accretive operators in a Banach space X. Applications, to the heat equation and wave equations, with time dependent boundary conditions, are given.

Introduction. The present paper gives a new existence result for the equation

$$x'(t) + A(t)x(t) \ni 0, \quad t \in [0, T]. \tag{P}$$

Here, for each $t \in [0, T]$, $A(t) : D(A(t)) \subset X \to 2^X$ is an m-ω-accretive operator in X. By X, we have denoted a Banach space, with uniformly convex dual X^*.

There are several papers referring to this subject. For the linear case, we mention the works of Kato-Tanabe [5] (analytic semigroups) and Kato [4]. In the nonlinear case, similar results can be found in [2], [3] and also in [6], [7].

The aim of this paper is to give general conditions allowing a more general t-dependence of $A(t)$, which guarantees the generation of an evolution operator for problem (P).

Roughly speaking the main idea behind our existence results, suggested by the problem itself, is that the most natural conditions to be imposed on $A(t)$ are those expressing the evolution of $t \mapsto A(t)$ only when t increases. This is the main difference between our results and those cited above, and as we shall see, our hypotheses are more simple, more general, and easy to check.

Another important difference between our paper and most of the previous papers is the way we prove our results. That is, we do not use the Crandall-Evans method (that is, by convergence of D.S. solutions) but the convergence of the solutions for the approximate problems

$$x'_\lambda(t) + A_\lambda(t)x_\lambda(t) = 0, \quad t \in [0, T],\ \lambda > 0. \tag{P$_\lambda$}$$

Here, A_λ is the Yosida approximate for A.

Section 1 includes the statement of our main results. A discussion of these results, in connection with other related results, may be found in Section 2. Section 3

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