Differential and Integral Equations, Volume 4, Number 1, January 1991, pp. 103–115.

ON MATHEMATICAL MODELS OF A VISCOELASTICITY WITH A MEMORY

V.P. Orlov and P.E. Sobolevskii

Department of Mathematics, Voronezh State University
Universitetskaya p1.1, 394693 Voronezh, USSR

(Submitted by: V.M. Matrosov)

Abstract. The boundary value problems describing a motion of viscoelastic mediums with a memory are considered. Local existence and uniqueness theorems are proved. The method of investigation is a reduction of the original problem to an operator equation in a space of summable functions. For solvability of the equation the principle of contracting transformations is used. This work continues the investigations of the mathematical models of the multidimensional mediums carried out in [8]. The reason for generalization of the results obtained in [8] was work [7] dedicated to the stationary viscoelastic mediums with a memory.

1. Reological relations. The motion of the continuous medium occupying the bounded volume $\Omega \subset \mathbf{R}^n$ $(n = 2, 3)$ with the invariable boundary $\partial\Omega \subset \mathbf{C}^2$ is being investigated. The motion equation in Cauchy form (see [1]) is

$$R[\partial v/\partial t + \mathcal{D}(v)] = \operatorname{Div} T_s + Rf, \qquad \mathcal{D}(v) = \sum_{k=1}^{n} v_k \partial v/\partial x_k. \tag{1}$$

Here $R(t,x)$ is the density of the medium, $v(t,x) = (v_1, \dots, v_n)$ is the velocity, $T_s(t,x) = \{t_{ij}\}_{i,j=1}^{n}$ is the stress tensor, $f(t,x)$ is the density of the exterior forces for a unit of mass in the time t and at the point x. By the sign $\operatorname{Div} A$, we imply a divergence of the matrix function $A(x)$; i.e., the column vector formed by divergences of the lines of the matrix $A(x)$. It is assumed that T_s depends on the tensor of viscous stresses T_s^v, on the tensor of elastic stresses T_s^e and on the spherical tensor of pressure pI ($p(t,x)$ is scalar). It is assumed also that T_s^v depends on the tensor of velocities of strains

$$T_{vs} = \frac{1}{2}[\partial v/\partial x + (\partial v/\partial x)^*] \tag{2}$$

(here $\partial v/\partial x$ is a matrix with the components $\partial v_i/\partial x_j$, the sign $*$ denotes an operation of conjugations of a matrix), and T_s^e depends on the strain tensor T_{st}. In order to define the strain tensor T_{st}, the Cauchy problem (in integral form)

$$z(\tau; t, x) = x + \int_t^\tau v(s, z(s; t, x))\, ds \quad (0 \le t \le t_0) \tag{3}$$

Received April 10, 1990.
AMS Subject Classifications: 35Q99.