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ON THE STABILITY OF THE ZERO SOLUTION OF A ONE-DIMENSIONAL MATHEMATICAL MODEL OF VISCOELASTICITY

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Abstract. The initial-boundary value problem for a mathematical model of a one-dimensional physical linear viscoelastic medium is considered. The Lyapunov's stability of the zero solution and, hence, nonlocal solvability for small initial data are established. First we prove a coercive solvability of some abstract differential equation of second order. Then by means of the abstract results obtained we reduce the original problem to an operator equation with a contractive operator.

1. Introduction. This work continues the investigations on the mathematical models of physical linear viscoelastic (multi-dimensional) and thermoelastic (one-dimensional) mediums carried out in [6–8]. The nonlinearities in the models above arose because of the difference between Lagrangian and Eulerian coordinates used for describing elasticity and viscosity respectively. In the works mentioned the local (in time) existence and uniqueness theorems were obtained.

Here we consider the initial-boundary value problem

$$u_{tt} - \mu_1 u_{xx} - \mu_2[(1+u_x)^{-1}u_{tx}]_x = f(t,x), \quad t \geq 0,\ 0 \leq x \leq 1,\ \ \mu_1, \mu_2 \geq 0; \quad (1)$$

$$u(0,x) = u_0(x),\ \ u_t(0,x) = u_1(x)\ \ (0 \leq x \leq 1), \quad u(t,0) = u(t,1) = 0\ \ (t \geq 0) \quad (2)$$

that describes in Lagrangian coordinates the motion of the one-dimensional viscoelastic medium (cf. [8]). We shall establish the Lyapunov's stability of the zero solution of problem (1)–(2) and, hence, the nonlocal solvability for "small" f, u_0 and u_1.

An important part in the proof of stability is an investigation of a coercive solvability and properties of some abstract differential equation of second-order in a Banach space. There is an extensive bibliography on second order abstract differential equations (cf. [3]). Peculiarity and goals in our case require a different approach to the solution of such equations.

2. Formulation of results. The solution of problem (1)–(2) is defined to be a function $u(t,x)$ having all (generalized) derivatives contained in the equation

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