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A VARIATIONAL APPROACH TO APPROXIMATION OF DELAY SYSTEMS

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Abstract. Using a varational formulation of the Trotter-Kato approximation theorem for strongly continuous semigroups we show that one can construct in a uniform way most of the approximation schemes for delay systems which have been published in recent years. In order to demonstrate that this approach is not restricted to delay systems we also construct two approximation schemes for size structured population models.

1. Introduction. Since 1974 (see [1]) a considerable number of approximation schemes for delay systems has been published, where the proof of convergence uses one or the other version of the Trotter-Kato theorem in semigroup theory (see for instance [1-3, 7-10, 12-14, 18]). In this paper we give a variational version of the Trotter-Kato theorem (Section 2) and show that this version can be used advantageously in order to prove convergence of the approximation schemes developed in the references mentioned above. In many cases the proof for consistency of a scheme is considerably simpler compared to the proof given originally. In Section 3 we give an outline of the convergence proofs for some of the algorithms already published. Since the approach taken in this paper is based on abstract convergence results for strongly continuous semigroups, the method is not restricted to delay systems. We demonstrate this by constructing two approximation schemes for size structured population models in Section 4.

2. A Trotter-Kato type theorem. Let V be a real Hilbert space with inner product $\langle\cdot,\cdot\rangle$ and corresponding norm $\|\cdot\|$. Furthermore, assume that $S(t)$, $t \geq 0$, is

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