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COMPARISON RESULTS FOR ELLIPTIC AND PARABOLIC EQUATIONS VIA SYMMETRIZATION: A NEW APPROACH

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Abstract. We present here a new approach for comparison results of solutions of second-order elliptic or parabolic equations by Schwarz symmetrization. This approach relies only on the fact that the fundamental solution of the heat equation in the whole space is spherically symmetric and decreasing and the proofs use, in addition to this well-known fact, a classical domination relationship and Trotter-Kato formula. We also apply this method to other symmetrizations like, for instance, Steiner symmetrization for which we thus derive some new comparison results.

I. Introduction. It is by now well known that sharp bounds for solutions of elliptic and parabolic equations may be obtained using Schwarz symmetrization (i.e., the spherical nonincreasing rearrangement). Indeed, for large classes of equations, the solution may be "compared" to the solution of an analogous problem with spherical symmetry (the so-called symmetrized problems). The first results in this direction were obtained by H. Weinberger [40], G. Talenti [36], C. Bandle [8] and since then, have been extended in various directions by various authors: let us mention for instance A. Alvino and G. Trombetti [5]; P.L. Lions [24]; G. Chiti [13]; J.L. Vasquez [39], C. Bandle [9]; J. Mossino and J.M. Rakotosson [31]; J.M. Rakotosson [33]; J.M. Rakotosson and R. Temam [34]; J. Mossino [30]; A. Alvino, P.L. Lions and G. Trombetti [1]; G. Talenti [37] (and the references therein). In the case of the "pure" Schwarz symmetrization, the most general results are presented in A. Alvino, P.L. Lions and G. Trombetti [2]. However, the proof of all these results relies on the same idea: obtain some differential inequality for the distribution function of the solution, an inequality which becomes an equality in the case of the symmetrized problem. Using the maximum principle, one obtains the desired

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