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LIAPUNOV OPERATORS AND STABILIZATION IN STRONGLY ORDER PRESERVING DYNAMICAL SYSTEMS

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Introduction. In this note we deal with the asymptotic behavior of (eventually) strongly monotone semigroups, $S(t)$, on strictly ordered Banach spaces. In our considerations, the continuity in t does not enter and so our results hold for t in $\mathbb{R}_0^+$ or $\mathbb{Z}_0^+$ alike; in particular, they hold for discrete semigroups generated by a single map. Moreover, no smoothness assumptions, besides the continuity of the maps $S(t)$ for fixed t, are introduced.

The paper addresses the following basic question: under which conditions are all relatively compact orbits convergent? Our answer is very simple and geometrical. For a semigroup $S(\cdot)$ as above, with initial conditions chosen from the order interval $[a, b]$, with a a subsolution, and b a supersolution, all precompact orbits are convergent if there is a continuous, strongly ordered arc Γ connecting a to b, and which, in general, may consist of two pieces Γ_1 and Γ_2, with the lower one made up of subsolutions and the upper one of supersolutions.

The most interesting, dynamically, is the case where (part of) Γ consists of a continuum of equilibria. In that case, Γ can be split into Γ_1 and Γ_2 in an infinity of ways. We point out that Γ need not be invariant under $S(\cdot)$, a feature that makes the result flexible in applications. We note that our structure hypotheses do not allow the existence of a non-degenerate unstable equilibrium on Γ except possibly at the end points. This feature, in general, is in the nature of things for stabilization of all precompact orbits to hold, for otherwise only generic results can be expected ([11], [12]).

The idea of the stabilization result is this: Given any element in the order interval $[a, b]$, there is either a maximal element of Γ_1 (subsolution) below it or a minimal element of Γ_2 (supersolution) above it. This fact, together with the strong monotonicity hypothesis, forces the ω-limit set into a single element, an equilibrium on Γ. In particular, Γ contains all the equilibria in the order interval $[a, b]$.

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