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SOLUTIONS OF A NONLINEAR ODE APPEARING IN THE THEORY OF DIFFUSION WITH ABSORPTION

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Abstract. We investigate the existence of solutions of the nonlinear ordinary differential equation $(f^m)'' - \beta\eta f' + \alpha f - f^p = 0$, which appears when we want to find selfsimilar solutions of the form $u(x,t) = t^{-\alpha} f(\eta)$, $\eta = xt^\beta$, for the nonlinear parabolic equation $u_t = (u^m)_{xx} - u^p$ in $Q = \mathbb{R} \times (0,\infty)$. We consider the range of exponents $1 < p < m$ where the existence and uniqueness theory for this equation has some peculiar features. The similarity exponents are (necessarily) $\alpha = \frac{1}{p-1}$, $\beta = \frac{m-p}{2(p-1)} = \frac{\alpha}{\gamma}$.

1. Introduction. We consider the nonlinear parabolic equation

$$u_t = (u^m)_{xx} - u^p \quad \text{in } Q = \mathbb{R} \times (0,\infty), \tag{1.1}$$

in which m and p are positive parameters. The equation appears in the theory of diffusion with absorption [2]. In this paper we investigate the existence and uniqueness of nonegative solutions to the Cauchy Problem for equation (1.1) in the exponent range

$$1 < p < m, \tag{1.2}$$

which offers a number of interesting phenomena. Thus, it has recently been shown in the paper [3] that equation (1.1) admits in the above range of exponents a solution $u(x,t) \in C(\overline{Q})$, $u \geq 0$ with initial data

$$u(x,0) = u_0(x) \in C(\mathbb{R}) \tag{1.3}$$

if we impose on the initial function a growth restriction of the type

$$0 \leq u_0(x) \leq c_0(k^2 + |x|^2)^{\gamma/2}, \tag{1.4}$$

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