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OSCILLATION PROPERTIES OF FIRST ORDER NONLINEAR FUNCTIONAL DIFFERENTIAL EQUATIONS OF NEUTRAL TYPE

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Abstract. We study the oscillatory and nonoscillatory behavior of solutions of neutral functional differential equations of the form

$$\frac{d}{dt}[x(t) + h(t)x(\tau(t))] \pm f(t, x(g_1(t)), \dots, x(g_N(t))) = 0, \tag{*}$$

assuming in particular that $h(t) > 0$, $\lim_{t\to\infty} \tau(t) = \infty$, $\lim_{t\to\infty} g_i(t) = \infty$, $1 \le i \le N$, and $y_1 f(t, y_1, \dots, y_N) \ge 0$ for $y_1 y_i > 0$, $1 \le i \le N$. First we obtain sufficient conditions under which all solutions of (*) are oscillatory, and then derive criteria for (*) to have bounded nonoscillatory solutions. As a result, we are able to indicate the existence of a class of nonlinear equations of the form (*) for which the situation of oscillation of all solutions can be completely characterized. The principal feature of this paper is that the following four cases for $h(t)$ and $\tau(t)$ are examined: $\{h(t) < 1,\ \tau(t) < t\}$, $\{h(t) < 1,\ \tau(t) > t\}$, $\{h(t) > 1,\ \tau(t) < t\}$, $\{h(t) > 1,\ \tau(t) > t\}$.

1. Introduction.

In this paper we are concerned with the oscillatory (and nonoscillatory) behavior of first order neutral functional differential equations of the form

$$\frac{d}{dt}[x(t) + h(t)x(\tau(t))] = f\big(t, x(g_1(t)), \dots, x(g_N(t))\big), \tag{A}$$

$$\frac{d}{dt}[x(t) + h(t)x(\tau(t))] + f\big(t, x(g_1(t)), \dots, x(g_N(t))\big) = 0, \tag{B}$$

for which the following conditions are always assumed to hold:

(a) $h : [t_0, \infty) \to (0, \infty)$ is continuous;

(b) $\tau : [t_0, \infty) \to \mathbb{R}$ is continuous and strictly increasing, and satisfies $\lim_{t\to\infty} \tau(t) = \infty$;

(c) $g_i : [t_0, \infty) \to \mathbb{R}$, $1 \le i \le N$, are continuous, and $\lim_{t\to\infty} g_i(t) = \infty$;

(d) $f : [t_0, \infty) \times \mathbb{R}^N \to \mathbb{R}$ is continuous; $f(t, y_1, \dots, y_N)$ is nondecreasing in each y_i, $1 \le i \le N$, and $y_1 f(t, y_1, \dots, y_N) \ge 0, \ne 0$ for $y_1 y_i > 0$, $1 \le i \le N$.

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