Differential and Integral Equations, Volume 4, Number 2, March 1991, pp. 411–423.

SPATIAL DECAY RESULTS FOR A CLASS OF QUASILINEAR ELLIPTIC EQUATIONS OF MODE ZERO

SHLOMO BREUER AND JOSEPH J. ROSEMAN
School of Mathematical Sciences, The Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, Ramat-Aviv, 69978, Israel

(Submitted by: A.R. Aftabizadeh)

1. Introduction. A classic problem presented to students in partial differential equations courses is to study solutions of Laplace's equation in a two-dimensional semi-infinite strip or three-dimensional cylinder, with homogeneous data on the lateral sides. By means of the separation of variables technique, they soon discover that if, in addition, it is assumed that the solutions do not grow exponentially at infinity, then they must decay exponentially with a minimal decay rate which depends upon the cross-section. For a two-dimensional strip of width h, the minimal decay rate constant is π/h.

Laplace's equation is the model of the linear second order elliptic type, and it may be obtained from the linearization of many nonlinear second order equations, for example, the class of equations

$$u_{,kk} = c_{ikmn} u_{,i}\, u_{,k}\, u_{,mn} \;, \tag{1.1)*}$$

where c_{ikmn} is constant, which, in particular, includes the minimal surface equation

$$(1 + u_{,i}\, u_{,i}\,)u_{,kk} - u_{,i}\, u_{,j}\, u_{,ij} = 0 \;. \tag{1.2}$$

Much investigation has been done, especially during the last twenty years, on the behavior at infinity of solutions of nonlinear elliptic equations in a two-dimensional strip or a three-dimensional semi-infinite cylinder, with homogeneous data of Dirichlet, Neumann, or mixed type on the lateral portion of the boundary. (cf. [1-6]). Motivated largely by the minimal surface equation (1.2), Roseman [1] considered (1.1) in a plane semi-infinite strip with homogeneous Dirichlet data on the long sides. He imposed the condition that the gradient of u be sufficiently (but not arbitrarily) small near infinity and that the second derivatives be bounded by one, from which he obtained the result that u and any number of its derivatives must decay exponentially, the specified number of derivatives as well as the rate of decay being dependent on the smallness of the assumed bound on the first derivatives. In [2], Knowles considered the same problem specifically for (1.2) in the

Received January 16, 1989.
AMS Subject Classifications: 35B40, 35B45, 35B50, 35J25, 35J60.

* The subscripts range over the values 1,2,3, unless it is specifically stated that they range only over the values 1 and 2. Differentiation with respect to x_k is denoted by a comma, and the summation convention is assumed.