Differential and Integral Equations, Volume 4, Number 2, March 1991, pp. 293–303.

# PROPAGATION OF SOLUTIONS OF A NONLINEAR FUNCTIONAL DIFFERENTIAL EQUATION

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(Submitted by: Glenn Webb)

**Abstract.** We study the propagation of the solutions of the functional differential equation $\dot{x}(t) = f(x(t)) + g(x_t)$, $x(0) = h \in X$, $x_0 \in L^p = L^p(-r, 0; X)$ by relating subspaces of $L^p \times X$ propagated by $T_A(t)$, the semigroup generated by the operator $A\{\phi, h\} = \{\phi', f(h) + g(\phi)\}$, $D(A) = \{\{\phi, h\}, \phi \in W^{1,p}(-r, 0; X), \phi(0) = h \in D(f)\}$ to subspaces of $X$ propagated by $T_f(t)$, the semigroup generated by $f$.

**0. Introduction.** In this paper we relate families of subsets propagated by $T_f(t)$, a semigroup of nonlinear operators generated by the operator $f$ in a Banach space $X$, to the families of subsets propagated in $L^p(-r, 0; X) \times X$ by the semigroup $T_A(t)$ generated by the operator

$$A\{\phi, h\} = \{\phi', f(h) + g(\phi)\}$$

$$D(A) = \{\{\phi, h\}; \phi \in W^{1,p}(-r, 0; X), \phi(0) = h \in D(f)\},$$

where $g$ is a function from $L^p(-r, 0; X)$ into $X$.

We also show that $T_A(t)\{\phi, h\}$ gives the integral solution of the functional differential equation

$$\dot{x}(t) = f(x(t)) + g(x_t), \quad x(0) = h, \ \ x_0 = \phi. \qquad \text{(FDE)}'$$

Thus, as $T_f(t)h$ is the integral solution of the differential equation

$$\dot{x}(t) = f(x(t)), \quad x(0) = h, \qquad \text{(DE)}$$

the propagation results on $T_A(t)$ and $T_f(t)$ relate the propagation of solutions of a delay differential equation and of the associated ordinary differential equation.

In [12] M. Reed, in order to study propagation of the solutions of a semilinear wave equation, introduces the following definition.

Let $\{Z_t\}_{t \geq 0}$ be a family of subsets of a Banach space $Z$. The family of operators $S(t)$, $t \geq 0$, propagates $\{Z_t\}$ if $S(t-s)Z_s \subset Z_t$ for all $0 \leq s \leq t$.

Received December 12, 1989, in revised May 7, 1990.
AMS Subject Classifications: 34K.