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OSCILLATIONS OF INTEGRO-DIFFERENTIAL EQUATIONS

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Abstract. We establish sufficient and also necessary conditions for certain linear and some nonlinear integro-differential inequalities and/or equations to have positive solutions. The following is a special corollary of our results.

COROLLARY. *Suppose that $K \in C[\mathbb{R}^+, \mathbb{R}^+]$ and let $T > 0$ be such that K is not identically zero on $[0, T]$. Then*

$$-\lambda + \int_0^\infty e^{\lambda s} K(s)\, ds \leq 0 \qquad \text{for some } \lambda > 0$$

is a necessary and sufficient condition for the integro-differential equation

$$\dot{x}(t) + \int_0^t K(t-s)x(s)\, ds = 0, \qquad t \geq T$$

to have a solution x which is positive on $[0, \infty)$.

1. Introduction. Consider the integro-differential inequality

$$\dot{y}(t) + \int_0^t K(t-s)y(s)\, ds \leq 0, \qquad t \geq T \tag{1}$$

where $T \geq 0$ and $K \in C[\mathbb{R}^+, \mathbb{R}^+]$. By a solution of (1), we mean a continuous function y which is defined for $t \geq 0$ and which satisfies (1) for $t \geq T$.

Among other things, we will prove in this paper that under the condition

$$-\lambda + \int_0^\infty e^{\lambda s} K(s)\, ds > 0 \qquad \text{for all } \lambda > 0, \tag{2}$$

(1) cannot have a solution y which is positive on $[0, \infty)$, while if $T > 0$ and K is not identically zero on $[0, T]$ and if

$$-\lambda + \int_0^\infty e^{\lambda s} K(s)\, ds \leq 0 \qquad \text{for some } \lambda > 0, \tag{3}$$

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