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ABSTRACT QUASILINEAR PARABOLIC EQUATIONS WITH VARIABLE DOMAINS

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Abstract. Local and global existence and uniqueness of strict solutions to quasilinear parabolic equations with variable operator domains but constant interpolation spaces are studied. Applications to quasilinear parabolic partial differential equations are also given.

1. Introduction. This paper is concerned with the local and global existence and uniqueness of solutions to the Cauchy problem in a Banach space X

$$\begin{cases} u'(t) = A(u(t))u(t) + f(u(t)), & 0 \le t \le T \\ u(0) = u_0 \end{cases} \tag{1.1}$$

where the function $f(\cdot)$ is defined on a proper subspace Y of X and, for all $x \in Y$, the operators $A(x) : D(A(x)) \to X$ generate an analytic semigroup in X. Moreover, we assume the existence of an interpolation space between $D(A(x))$ and X, independent of x. Quasilinear parabolic equations with constant domains have been studied by several authors (among them, we quote Sobolevskii [36], Friedman [19], Poitier-Ferry [34], Lunardi [30, 31], Amann [7]) using a linearization method and regularity properties of the solutions of linear equations. Also in the present case, we want to solve problem (1.1) using a linearization method; i.e., we try to find a suitable function space Z such that there is a fixed point in Z of the application $T : Z \to Z$, $Tv = u$, where u is the solution of the linearized problem:

$$\begin{cases} u'(t) = A(v(t))u(t) + f(v(t)), & 0 \le t \le T \\ u(0) = u_0. \end{cases} \tag{1.2}$$

Therefore we need existence and regularity results for the solution of the linear parabolic equation

$$\begin{cases} u'(t) = A(t)u(t) + f(t), & 0 \le t \le T \\ u(0) = u_0 \end{cases} \tag{1.3}$$

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