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AN APPLICATION TO BIFURCATION PROBLEMS OF CUSPS IN BANACH SPACES

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1. Introduction and statement of the results. Let X, Y, B be Banach spaces, $f : B \times X \to Y$ a C^3 function such that for each $\lambda \in B$, $f(\lambda, 0) = 0$. In this paper we want to show how to deduce some results on bifurcation for f by means of the geometrical information obtained by singularity theory. More precisely, we define a map $\Phi : B \times X \to B \times Y$ as

$$\Phi(\lambda, x) := (\lambda, f(\lambda, x));$$

we observe that

$$f(\lambda, x) = 0 \quad \text{if and only if} \quad \Phi(\lambda, x) = (\lambda, 0), \tag{0}$$

and we assume $(\lambda^*, 0)$ to be a cusp point for Φ. Therefore f satisfies, at $(\lambda^*, 0)$, the following conditions (see [1] and [4]):

1. $\dim \operatorname{Ker} f'_x(\lambda^*, 0) = 1$, $\operatorname{Im} f'_x(\lambda^*, 0)$ is closed, codim $\operatorname{Im} f'_x(\lambda^*, 0) = 1$.
2. $\langle f''_{xx}(\lambda^*, 0)[v_0][v_0], \gamma_0 \rangle = 0$, where $\gamma_0 \in Y^* \backslash \{0\}$[1] is such that $\operatorname{Ker} \gamma_0 = \operatorname{Im} f'_x(\lambda^*, 0)$ and v_0 is a vector generating $\operatorname{Ker} f'_x(\lambda^*, 0)$.
3. There exists a $(\hat{\mu}, \hat{v}) \in B \times X$ such that

$$\langle f''_{xx}(\lambda^*, 0)[v_0][\hat{v}], \gamma_0 \rangle + \langle f''_{\lambda x}(\lambda^*, 0)[v_0][\hat{\mu}], \gamma_0 \rangle \neq 0. \tag{H_1}$$

4. $\langle f'''_{xxx}(\lambda^*, 0)[v_0][v_0][v_0], \gamma_0 \rangle + 3 \langle f''_{xx}(\lambda^*, 0)[v_0][\Gamma v_0], \gamma_0 \rangle \neq 0$, where $\Gamma z = -f'_x(\lambda^*, 0)^{-1} \left(f''_{xx}(\lambda^*, 0)[z][v_0] \right)$.

Our goal is to show in which cases the fact that $(\lambda^*, 0)$ is a cusp point for Φ implies that $(\lambda^*, 0)$ is a bifurcation point for the map f, and in such a case to describe the precise behaviour of the branch of bifurcation. Note that the bifurcation parameter λ can lie in any Banach space B.

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[1] X^* denotes the dual of the Banach space X.