ON A CLASS OF SINGULAR OR DEGENERATE
HYPERBOLIC VARIATIONAL INEQUALITIES†

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Abstract. A class of hyperbolic variational inequalities is studied, where the unilateral constraints concern the time derivative of the unknown function; singularities or degeneracies of various kinds, with respect to the time variable, are considered. Some abstract existence and uniqueness results are proved, in the framework of suitable weighted spaces. Various applications are also given.

1. Introduction. It is well known that various kinds of hyperbolic variational inequalities arise in several applications from Mechanics and Physics: see, e.g., the book by Duvaut and Lions [16], with its bibliography. (We also refer to the papers by Amerio and Prouse [2], Amerio [1], Citrini [12], Schatzman [26], along with their bibliographies (and to various subsequent papers by the same authors), for other important unilateral problems for hyperbolic partial differential equations).

Moreover, some general "abstract" existence and uniqueness results are well known for hyperbolic variational inequalities, when the unilateral constraints concern the time derivative of the unknown function; see, in particular, Lions [18], Brezis [10].

On the other hand, various kinds of singular or degenerate linear hyperbolic partial differential equations also arise in several applications. Let us mention, e.g., the Euler-Poisson-Darboux equation, the Tricomi equation (considered in the hyperbolic region), the Euler-Fuchs equation: remark that such examples concern, in particular, singularities or degeneracies with respect to the time variable.

For singular or degenerate hyperbolic (and also parabolic) equations, the main reference up to 1976 is the book by Carroll and Showalter [11], with its large bibliography. For further studies and results on singular or degenerate hyperbolic equations (in particular, from the point of view of abstract differential equations)