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EXISTENCE THEOREMS FOR FOCAL BOUNDARY VALUE PROBLEMS

S. UMAMAHESWARAM AND M. VENKATA RAMA†
School of Mathematics and Computer Information Sciences
University of Hyderabad, Central University P.O., Hyderabad – 500 134, India

(Submitted by: A.R. Aftabizadeh)

Abstract. Sufficient conditions in terms of f and some auxiliary functions $u(x)$, $v(x)$ are given for the existence of a solution of the 2-point right focal boundary value problem $y^{(n)} = f(x, y, \ldots, y^{(n-1)})$, $y^{(i)}(x_1) = y_{1i}$, $i = 0, \ldots, m-1$, $y^{(i)}(x_2) = y_{2i}$, $i = m, \ldots, n-1$ where $1 \leq m < n$ is an arbitrary integer and $x_1 < x_2$, y_{1i}, y_{2i} are arbitrary real numbers. An alternative set of sufficient conditions entirely in terms of f are also given for the above boundary value problem.

1. Introduction. We are interested in the differential equation

$$y^{(n)} = f(x, y, \ldots, y^{(n-1)}) \tag{1.1}$$

along with "2-point right focal" boundary conditions (BC's)

$$\begin{cases} y^{(i)}(x_1) = y_{1i}, & i = 0, \ldots, m-1 \\ y^{(i)}(x_2) = y_{2i}, & i = m, \ldots, n-1 \end{cases} \tag{1.2}$$

where $n > 1$ is a fixed positive integer, f is continuous $I \times \mathbb{R}^n$ ($I \subset \mathbb{R}$ an interval), $1 \leq m < n$ is an arbitrary integer and $x_1, x_2 \in I$ ($x_1 < x_2$), y_{1i}, y_{2i} are arbitrary real numbers.

There are only a few theorems in the literature which give sufficient conditions for the existence of a solution of the "k-point right focal" boundary value problem (BVP) (1.1) and

$$y^{(i)}(x_r) = y_{ri}, \quad i = s(r-1), \ldots, s(r)-1; \qquad r = 1, \ldots, k \tag{1.3}$$

where k ($1 < k \leq n$), $n(1), \ldots, n(k)$ are arbitrary but fixed integers; $s(0) = 0$, $s(r) = n(1) + \cdots + n(r)$, $r = 1, \ldots k$, $s(k) = n$, $x_r \in I$ ($x_1 < \cdots < x_k$) and y_{ri} are arbitrary real numbers.

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