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ON SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS WITH MEASURES AS CONTROLS

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Abstract. We study a general class of nonlinear systems of ordinary differential equations, whose right-hand sides depend linearly on a vector valued measure. This measure is viewed as the distributional derivative of a control function with bounded variation. We discuss existence, uniqueness and continuous dependence of the solutions on the controls. Our interest in this kind of differential systems is mainly motivated by some recent applications of control theory in rational mechanics (see [8-11], [19], [21-22]).

1. Introduction. Given $m+1$ vector fields $f, g_1, \cdots, g_m$ from $\mathbb{R}^n$ into $\mathbb{R}^n$, we consider the Cauchy problem

$$\dot{x}(t) = f(x(t)) + \sum_{i=1}^{m} g_i(x(t))\dot{u}^i(t), \quad x(0) = \bar{x}, \tag{E}$$

where the control function $u = (u^1, \cdots, u^m) : [0,T] \to \mathbb{R}^m$ enters in the equation only through its derivative $\dot{u}$. Note that the case where f and the g_i depend also on t and u may be reduced to the form (E) by adding the variables $x^0 = t$, $x^{n+i} = u^i$, and the corresponding equations $\dot{x}^0 = 1$, $\dot{x}^{n+i} = \dot{u}^i$. We shall always assume that the vector field $f, g_1, \cdots, g_m$ are globally bounded and continuously differentiable on $\mathbb{R}^n$. Therefore, if u is continuously differentiable, then problem (E) has a unique classical (continuously differentiable) solution. If u is absolutely continuous on $[0,T]$, then problem (E) has a unique absolutely continuous solution in the sense of Carathéodory.

The aim of this paper is to study the Cauchy problem (E) when u is just a function with bounded variation on $[0,T]$. The case where the functions g_i depend merely on t was throughout investigated in [23]. Actually, the main difficulty in our situation lies in the fact that the g_i depend on the state variable x. Recently, some interest has been devoted to certain evolution problems which involve functions with bounded variation in time (see e.g., [3-11], [14-17]). In fact, such maps appear quite naturally in the description of some physical processes suffering impulses.

In particular, we wish to mention a recent application in rational mechanics (see [8-11], [19], [20-21]) which involves a differential system of the form of (E),

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