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NULL CONTROLLABILITY OF FIRST ORDER QUASILINEAR EQUATIONS

VIOREL BARBU†
Faculty of Mathematics, University of Iasi, 6600 Iasi, Romania

Abstract. It is shown that the first order quasilinear equation $y_t + \sum_{i=1}^{N}(a_i(y))_{x_i} = u$ in $\mathbb{R}^N \times \mathbb{R}^+$, $y(x,0) = y_0(x)$ in $\mathbb{R}^N$ with control constraint $\|u(t)\|_{L^1(\mathbb{R}^N)} \leq \rho$ is null controllable in finite time.

1. Introduction. Consider the controlled system

$$y_t + \sum_{i=1}^{N}(a_i(y))_{x_i} = u \quad \text{for } t > 0, \quad x \in \mathbb{R}^N, \tag{1.1}$$
$$y(x,0) = y_0(x) \quad \text{for } x \in \mathbb{R}^N,$$

where $a = (a_1, a_2, \dots, a_N) : \mathbb{R} \to \mathbb{R}^N$ is a continuous function satisfying the condition

$$\lim_{|\gamma|\to 0} \|a(\gamma)\|/|\gamma| < \infty, \tag{1.2}$$

$y_0 \in L^1(\mathbb{R}^N)$ and $u \in L^1(0,T;L^1(\mathbb{R}^N))$, $\forall T > 0$. We have denoted by y_t and y_{x_i} the partial derivatives of $y : \mathbb{R}^N \times (0,\infty) \to \mathbb{R}$ with respect to t and x_i.

M.G. Crandall [5] has shown that (1.1) can be treated as an abstract Cauchy problem

$$\frac{dy}{dt} + Ay = u \quad \text{in } [0,\infty), \tag{1.3}$$
$$y(0) = y_0$$

in the space $X = L^1(\mathbb{R}^N)$, where A is m-accretive in X. More precisely, A is the closure in $X \times X$ of the operator A_0 defined by $y \in D(A_0)$, $w \in A_0 y$ if $y, w \in L^1(\mathbb{R}^N)$ and

$$\int_{\mathbb{R}^N} \operatorname{sign}_0(y(x) - k)\,[(a(y(x)) - a(k),\, \varphi_x(x)) + w(x)\varphi(x)]\,dx \geq 0 \tag{1.4}$$

for every $\varphi \in C_0^\infty(\mathbb{R}^N)$ and such that $\varphi \geq 0$ and every $k \in \mathbb{R}$ (here $(\cdot,\cdot)$ is the usual scalar product in $\mathbb{R}^N$). Then by standard existence theory for nonlinear evolution

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