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NECESSARY AND SUFFICIENT STABILITY CONDITIONS IN AN ASYMPTOTICALLY ORDINARY DELAY DIFFERENTIAL EQUATION

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Abstract. In this paper, we investigate the stability and asymptotic stability of the zero solution of a delay differential equation with small time dependent delay. Our main result shows that if the delay is small enough, then the zero solution of the delay equation is stable (asymptotically stable) if and only if the zero solution of some related ordinary differential equation is stable (asymptotically stable). The proofs of our results are based on the fact that the investigated delay equation is asymptotically ordinary; that is, the delay is so small that the infinite dimensional space of the solutions of the delay equation is asymptotically equivalent to a finite dimensional space.

1. Notations and main results.

Consider the delay differential equation

$$\dot{x}(t) = Ax(t-\tau(t)), \tag{1.1}$$

where $A \in \mathbb{R}^{n\times n}$ is a given n by n matrix with real entries and $\tau\colon \mathbb{R}_+ \to \mathbb{R}_+$ ($\mathbb{R}_+ = (0,\infty)$) is a continuous function and

$$r(t_0) = \sup_{t\geq t_0} \tau(t) < \infty, \quad t_0 \geq 0. \tag{1.2}$$

The initial condition associated to equation (1.1) is

$$x(t) = \varphi(t), \quad t_0 - r(t_0) \leq t \leq t_0, \tag{1.3}$$

where $t_0 \geq 0$ and $\varphi \in C_{t_0} \equiv C([t_0 - r(t_0), t_0], \mathbb{R}^n)$. It is well known ([2], [7]) that for any $\varphi \in C_{t_0}$, $t_0 \geq 0$, equation (1.1) has one and only one solution on $[t_0 - r(t_0), \infty]$ with initial condition (1.3). The solution of (1.1) and (1.3) is denoted by $x(t_0, \varphi)(t)$, $t \geq t_0 - r(t_0)$.

The norm of a vector $x \in \mathbb{R}^n$ is denoted by $|x|$ and the norm of a matrix $A \in \mathbb{R}^{n\times n}$ is defined by $\|A\| = \sup\{|Ax| \colon x \in \mathbb{R}^n, |x| = 1\}$.

In this paper, we give some necessary and sufficient conditions for the zero solution of equation (1.1) to be stable and asymptotically stable. In our investigations,

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