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REMARKS ON RESONANCE PROBLEMS WITH UNBOUNDED PERTURBATIONS

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Abstract. We consider a class of unbounded perturbations of a linear resonant problem with Dirichlet and Neumann boundary conditions and use elementary variational methods to show the existence of a solution.

1. Introduction and statement of results. Let Ω be a bounded smooth domain of $\mathbb{R}^N$, $N \geq 1$. We are concerned with the nonlinear resonance problem

$$\Delta u + \lambda u + g(x,u) = h(x) \quad \text{in } \Omega \tag{1}$$

with Dirichlet boundary condition

$$u = 0 \quad \text{on } \partial\Omega \tag{2}$$

or Neumann boundary condition

$$\frac{\partial u}{\partial n} = 0 \quad \text{on } \partial\Omega. \tag{3}$$

Here λ is an eigenvalue of $-\Delta$ in Ω with boundary conditions (2) or (3), $h(x) \in L^2(\Omega)$ and $g : \Omega \times \mathbb{R} \to \mathbb{R}$ is a nonlinear term satisfying hypotheses to be specified below.

Throughout the paper it is assumed that $g(x,s)$ is a Caratheodory function; i.e., $g(\cdot,s)$ is measurable on Ω for each $s \in \mathbb{R}$ and $g(x,\cdot)$ is continuous on $\mathbb{R}$ for almost every $x \in \Omega$. Assuming some growth and sign conditions on $g(x,s)$, we search for (weak) solutions of problems (1)-(2) or (1)-(3) as critical points in $H_0^1(\Omega)$ or $H^1(\Omega)$, respectively, of the functional

$$f(u) = \int_\Omega \left(\frac{1}{2}|\nabla u|^2 - \frac{1}{2}\lambda u^2 - G(x,u) + h(x)u \right) dx,$$

where $G(x,s)$ denotes the primitive $\int_0^s g(x,t)dt$ (the growth condition assumed below on $g(x,s)$ will ensure that f is C^1).

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