

## PERTURBING SEMIGROUPS BY SOLVING STIELTJES RENEWAL EQUATIONS

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Dedicated to George Maltese on the occasion of his 60th birthday

(Submitted by: Glenn Webb)

**Abstract.** We develop a perturbation theory for strongly continuous semigroups and dual semigroups not based on perturbation of infinitesimal generators but on certain families of bounded linear operators describing the cumulative effect of the feedback. The theory extends the theory of perturbation of generators by bounded or relatively bounded linear operators. The theory is applied to problems of structured population dynamics which cannot, to the best of our knowledge, be treated using a more conventional perturbation theory.

**1. Introduction.** Consider a linear dynamical system whose state space  $X$  is a Banach space. Assume that the *output* of the system can be described by a (bounded) linear map  $B$  from  $X$  into another Banach space  $Y$ . So, when  $t \mapsto u(t) \in X$  describes the time evolution of the state, the map  $t \mapsto Bu(t) \in Y$  gives the output as a function of time.

The *cumulative output* up to time  $t$  is, by definition,  $\int_0^t Bu(s)ds$ . For autonomous systems there exists a semigroup of operators  $T_0(t)$  on  $X$  such that  $u(t) = T_0(t)x$ , where  $x$  denotes the initial state at time  $t = 0$  and we can define cumulative output maps, parameterized by  $t$ , by

$$V_0(t)x = \int_0^t BT_0(s)xds.$$

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