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INTERIOR AND BOUNDARY REGULARITY OF THE WAVE EQUATION WITH INTERIOR POINT CONTROL

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Abstract. We study interior and boundary regularity of wave equations defined on an open bounded domain Ω, $\dim\Omega = 1, 2, 3$, and subject to the action of point control (through the Dirac distribution δ) at an interior point of Ω. A general approach is used which applies to other dynamics as well (e.g., Euler-Bernoulli equations, Kirchhoff equations, Schrödinger equations). For waves (and Kirchhoff) equations the interior regularity results are "$1/2+\varepsilon$" sharper in space Sobolev regularity over those that can be obtained by simply using that $\delta \in [H^\alpha(\Omega)]'$, $\alpha = 3/2+\varepsilon$, $1+\varepsilon$, $1/2+\varepsilon$, for $N = 3, 2, 1$. The boundary regularity results are "1/2" sharper in space Sobolev regularity over those that can be inferred by applying (formally) trace theory to the corresponding interior results.

1. Introduction, statement of the problem, preliminaries.

1.1. Statement of the problem and literature. Let Ω be an open, bounded domain in $\mathbb{R}^N$ ($N = 1, 2, 3$) with sufficiently smooth boundary $\Gamma = \partial\Omega$. In this paper we study interior and boundary regularity of wave equations defined on Ω with homogeneous boundary conditions (of various type) on Γ, and subject to the action of point control exercised at an interior point of Ω. As a matter of fact, the specific case of interior regularity of the wave equation with Dirichlet (homogeneous) boundary conditions in the case of $N = \dim\Omega = 3$ has already been studied, with three different proofs reported in J.L. Lions [6]: one proof, due to Y. Meyer [16], uses harmonic analysis; another proof, due to L. Nirenberg uses propagation properties of waves in $\mathbb{R}^3$, in particular the explicit classical Kirchhoff formula for the solution of the Cauchy problem in $\mathbb{R}^3$ (essentially the same proof appears in [21]); a third due to J.L. Lions uses a recent trace property for the normal derivative of the corresponding homogeneous problem [7], [11, 12], [8]. Some proofs (e.g., Nirenberg's) refer to the dual problem z in (2.7)). Instead, our proof centers on the original problem w in (2.1). Our proof follows a very general approach, which applies to any dynamics [in particular, waves, Euler-Bernoulli (plate) equations, Kirchhoff (plate) equations, Schrödinger equations, [17], [18], regardless of whether or not they possess finite speed of propagation] and in principle to any dimension $N = \dim\Omega$ (we confine our attention to $N = 1, 2, 3$). The key steps are:

(i) the analysis of (sharp) regularity of the corresponding free space problem by means of Laplace-Fourier transform techniques, which hinges on a sharp *a-priori* estimate (in the present case of wave problems, this estimate, Lemma

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