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BLOW-UP OF POSITIVE SOLUTIONS OF A NON-COOPERATIVE PARABOLIC SYSTEM†

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In recent years, a big effort has been devoted to the the study of the occurrence of blow-up of positive solutions of semilinear parabolic problems. See [3], [6], [7], [8], [10], [15] and the references therein.

A little experience on this subject shows that in treating these kinds of problems, the situation can change radically passing from a single equation to a system. This is due to several facts: lack of maximum or other invariance principles, absence of variational (or Lagrangian) structure of the associated stationary problem, complexity of the stationary solution set, etc.

In this paper, we shall consider a special system of equations of the form

$$\begin{aligned} &\vec{u}_t = \Delta\vec{u} + A\vec{u} + f(\vec{u}) && \text{in } \Omega\times[0,+\infty),\\ &\vec{u} = 0 && \text{on } \partial\Omega\times[0,+\infty),\\ &\vec{u}(x,0)=\vec{u}_0(x) && \text{in } \Omega, \end{aligned} \tag{P}$$

where Ω is a smooth bounded domain contained in $\mathbb{R}^n\,(n\geq 1)$, A is a constant 2×2 matrix, f is a given superlinear function and $\vec{u}_0$ is a componentwise positive function.

If (P) is cooperative, i.e., the off-diagonal entries of A are nonnegative, and f has cooperative Jacobian, i.e., $\frac{\partial f_i}{\partial x_i}\geq 0$ if $i\neq j$, similar problems have been already studied in the literature; see [5], [3] and the references therein.

One of the main features of this paper is that we do not assume that (P) is cooperative. In this case, it is fairly easy to show, by using classical results of semigroup theory (see [11]), that the whole positive cone of the state space *cannot* be invariant for the associated flow. However, due to the special structure of our problem (see (jv) of Section 1 and (i) of Theorem 1.4), it is possible to transform (P) into an equivalent 3×3 cooperative system and to use the machinery of the maximum principle for cooperative problems (see, [12]).

The plan of the paper is the following. In Section 1, we consider some preliminary results which are essential for the sequel. Section 2 contains the study of the blow-up of solutions of (P) when Ω is a general domain by using two classical methods, namely, the energy and eigenfunction method. Section 3 deals with symmetric

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