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ON A NONLINEAR SECOND ORDER DIFFERENTIAL EQUATION WITH OSCILLATING COEFFICIENT

M. CECCHI AND M. MARINI
Facolta' di Ingegneria, Univ. di Firenze, Via di S. Marta, 3-50139 Firenze, Italy

GAB. VILLARI
Facolta' di Scienze, Univ. di Firenze, Viale Morgagni 67/a, 50134 Firenze, Italy

(Submitted by: G. Da Prato)

1. Consider the nonlinear equation

$$(p(t)x'(t))' + q(t)f(x(t)) = 0 \qquad \left(' = \frac{d}{dt}\right), \tag{E_f}$$

where p, $q : [0, +\infty) \to \mathbb{R}$ and $f : \mathbb{R} \to \mathbb{R}$ are continuous, $p(t) > 0$, and $u \cdot f(u) > 0$ for $u \neq 0$.

When $q(t) < 0$, it is well known (see, e.g., [15])) that all nontrivial solutions $x = x(t)$ of (E_f) which are defined on $[\alpha_x, +\infty)$, $\alpha_x \geq 0$, are nonoscillatory. Further, the set of solutions of (E_f) may be divided into the following two classes:

$$A = \{x, \text{ solution of } (E_f) \ : \exists t_x \geq \alpha_x : x(t) \cdot x'(t) > 0 \quad \text{for } t > t_x\},$$

$$B = \{x, \text{ solution of } (E_f) \ : x(t) \cdot x'(t) < 0 \quad \text{for } t \geq \alpha_x\}.$$

Solutions in class A are eventually either positive increasing or negative decreasing, while solutions in class B are either positive decreasing or negative increasing.

The continuability and the asymptotic behavior of solutions of (E_f) in class A, when $q(t) < 0$, have been studied in [15], extending some results from the corresponding linear equation

$$(p(t)x'(t))' + q(t)x(t) = 0. \tag{E_l}$$

As for class B, it is shown in [13] that there exist equations of the type (E_f) without solutions in class B as in the case of the equation $x'' = |x|^{1/2}$. On the other hand, if q can have negative values and f satisfies a certain growth condition, then there always exist solutions of (E_f) which are noncontinuable [2]. As for the existence of continuable solutions of class B, if conditions on f guarantee existence, uniqueness and continuous dependence on initial values, then (E_f) has at least a solution in class B, as may be proved using a classical argument of A. Mambriani

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