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GENERALIZED SOLUTIONS IN A NEW TYPE OF SETS FOR PROBLEMS WITH MEASURES AS DATA

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Introduction. Consider Ω an open bounded set of $\mathbb{R}^N$ and let u be a solution of $-\text{div}\,(|\nabla u|^{p-2}\nabla u) = \delta_{x_0}$, where $x_0 \in \Omega$, $1 < p \leq N$.

If we want to find a solution u in a Sobolev space $W_0^{1,q}(\Omega)$, $q > \max(1, p-1)$, then one can easily check that necessarily $q < \frac{N}{N-1}\,(p-1)$; this implies that $p > 2 - \frac{1}{N}$ (see [10], [11], [12], [2]).

A natural question is: what kind of solutions do we have if $p \leq 2 - \frac{1}{N}$? I gave a first answer to that question in [13, 14] by introducing a class of solutions in which the problem is well posed. But, I only solved the case where the data are in $L^1(\Omega)$. P.L. Lions and F. Murat [9] also solve problems with L^1-data using the concept of renormalized solution introduced by DiPerma-Lions. P. Benilan ([1]) also treated the p-Laplacian with L^1-data considering similar sets as we introduce here. In [9] and [1], they get an uniqueness result.

The main "difficulty" in solving problems with measures was getting an a priori estimate in some Lebesgue space $L^\gamma(\Omega)$, $\gamma > p-1$. Here, this difficulty is overcome by the use of some property of a new type of sets. The proof of the estimate is inspired by a recent paper of T. Kilpeläinen and J. Màly ([7], 1990).

The compactness result produced in the preceding papers is available for any case. So, the main purpose of this paper is to give a complete resolution of the problem $-\text{div}\,(a(x, u, Du)) = \mu$ (bounded Radon measures) when $a(x, u, \xi) \geq \alpha|\xi|^p$, $\alpha > 0$ and $1 < p \leq 2 - \frac{1}{N}$. It is clear that the method that is provided here works for the generalized Leray-Lions operators introduced in [14].

This work completes those results done by [2], [3], [4], [5], [6] (also see the references therein).

The organization of our paper will be the following.

I.) Definition of a generalized solution: New type of sets — main theorem.

II.) Proof of the main theorem: Approximate problems and priori estimates.

III.) Recalling the compactness result.

IV.) Passing to the limit and various remarks.

I.) Definition of a generalized solution: Notion of T-sets and main theorem.

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