Differential and Integral Equations, Volume 4, Number 6, November 1991, pp. 1391–1417.

SINGULAR THIRD-ORDER BOUNDARY VALUE PROBLEMS

DONAL O' REGAN
Department of Mathematics, Maynooth College, Co. Kildare, Ireland

(Submitted by: A.R. Aftabizadeh)

Abstract. Methods based on Topological Transversality are used to derive existence theorems for a variety of singular third-order boundary value problems of the form $y''' + f(t,y,y',y'') = 0$, $0 < t < 1$. Here f may be singular at $t = 0$, $t = 1$, $y = 0$, $y' = 0$ and/or $y'' = 0$.

1. Introduction. This paper presents existence results for solutions to (singular) third-order boundary value problems of the form

$$y''' + f(t,y,y',y'') = 0, \quad 0 < t < 1; \ \ y \in B$$

where B specifies suitable boundary conditions. Most of the available literature, for example [1], [5], [8], [10] and [12], discuss the case when $f : [0,1] \times \mathbb{R}^3 \to \mathbb{R}$ is continuous, however in this paper we will allow our nonlinear term f to be singular at either $t = 0$, $t = 1$, $y = 0$, $y' = 0$ and/or $y'' = 0$. Many mathematical models which arise for example in boundary layer theory , reaction diffusion equations and non-Newtonian fluid theory are governed by singular second-order boundary value problems; see for example [2], [3], [4], [15] and [16]. As a result, singular second-order problems have become quite popular in the last few years; however, the examination of singular third- (and higher) order boundary problems is a relatively new and unexplored area. For this paper we will allow B to be the set of functions which satisfy either

(a) $y''(1) = c \geq 0$, $y'(0) = b \geq 0$, $y(0) = a \geq 0$
(b) $y'(1) = c \geq 0$, $y'(0) = b \geq 0$, $y(0) = a \geq 0$ or
(c) $y''(0) = 0$, $y(0) = a \geq 0$, $y(1) = b \geq 0$.

We remark here that throughout this paper we will treat in detail the case where B denotes boundary condition (a) whereas in the cases where B represents the other boundary conditions we will provide the details only where it is necessary. Many other boundary conditions could be considered; however, since the analysis is similar in these cases we choose to omit the details.

2. The equation $\mathbf{y''' + f(t,y) = 0}$. The author in [13] discusses problems of this type where the nonlinear term f is singular at $y = 0$. However, here we will allow our nonlinear term f to be singular at $t = 0$, $t = 1$ and/or $y = 0$, so for example terms like $f(t,y) = t^{-\alpha}(1-t)^{-\beta}y^{-\gamma}$, $0 \leq \alpha, \beta, \gamma < 1$ will be included in the results

Received September 1988, in revised April 1989.
AMS Subject Classifications: 34B15.