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GLOBAL BIFURCATION ANALYSIS OF AN ADAPTIVE CONTROL SYSTEM†

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Abstract. We present complete bifurcation analysis of a 2-parameter, 2-dimensional quadratic o.d.e. which arises in the study of Model Reference Adaptive Control (MRAC) systems. The 2-dimensional o.d.e. exhibits saddle-node, (subcritical) Hopf, and saddle-loop (a.k.a. homoclinic connection) bifurcations. We have been able to completely describe the bifurcation diagram by identifying all the bifurcation curves. All but the saddle-loop bifurcation curve have been explicitly characterized. The saddle-loop bifurcation curve, however, has been described qualitatively and its end-points as well as its relative position have been identified.

1. Introduction. This study concerns the global bifurcation analysis of a 2-dimensional quadratic ordinary differential equation (o.d.e.) which arises in the study of Model Reference Adaptive Control (MRAC) systems [11]. Specifically, the 2-dimensional o.d.e. captures the essential dynamics that a simple adaptive control system may generate in the presence of external disturbances. The dynamics of the equation were first characterized via digital computer simulations in [1] and were found to exhibit various types of bifurcations for a *specific set of parameters.* Due to the limitation of the computational accuracy of computers, it is essential to verify the bifurcations suggested by the (computer) simulations by analytical means. Moreover, analysis may provide explicit characterization of bifurcation curves and may subsequently lead to the construction of the complete global bifurcation diagram of the system analytically. Such analytical studies do provide support to computer simulations and could bring new insights into alternative choices of system parameters in the design stage of MRAC systems.

This paper is based on the conference version which appeared in [11]. It is organized as follows: In Section 1, we briefly describe the formulation in Model Reference Adaptive Control (MRAC) systems, then we specialize it to the prototype MRAC

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