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SEMIGROUP AND INTEGRAL FORM OF A CLASS OF PARTIAL DIFFERENTIAL EQUATIONS WITH INFINITE DELAY

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Dedicated to Professor Y.X. Xiao on his Career
of Mathematics Education for 50 years

Abstract. In this paper a class of partial differential equations with infinite delay is transformed into a neutral functional differential equation on so called friendly admissible phase spaces. Existence and uniqueness of solutions are considered in a semigroup-theoretical setting of unbounded operators. The equivalence between the considered differential equations and some integral equation is studied. A representation of solutions is given in phase spaces by Laplace transformation.

1. Introduction. The main purpose of this paper is to study the following partial differential equations with infinite delay:

$$\begin{aligned}
&\frac{\partial}{\partial t}\Big[u(\eta,t)-\sum_{j=1}^{\infty}b_j(\eta)\int_0^1 d\mu_j(\theta)u(\theta,t-r_j)-\int_{-\infty}^{t}g(t-s)b(\eta)\int_0^1 d\mu(\theta)u(\theta,s)ds\Big]\\
&=\frac{\partial^2}{\partial\eta^2}u(\eta,t)+a_0u(\eta,t)+\sum_{j=1}^{\infty}a_ju(\eta,t-r_j)+\int_{-\infty}^{t}h(t-s)u(\eta,s)ds,\\
&\qquad\qquad\text{for } 0<\eta<1 \text{ and } t\ge 0,\\
&\quad u(0,t)=u(1,t)=0,\quad t\ge 0,\\
&\quad u(\eta,\theta)=\varphi(\eta,\theta),\ 0\le\eta\le 1,\quad \theta\le 0,
\end{aligned}$$

which can be transformed into the following neutral functional differential equation with infinite delay:

$$\begin{aligned}
&\frac{d}{dt}\Big[x(t)-\sum_{j=1}^{\infty}B_jx(t-r_j)-\int_{-\infty}^{t}G(t-s)x(s)ds\Big]\\
&\quad=Ax(t)+A_0x(t)+\sum_{j=1}^{\infty}A_jx(t-r_j)+\int_{-\infty}^{t}H(t-s)x(s)ds.
\end{aligned}$$

In finite dimensions, neutral equations with infinite delay have been studied in several papers (see, cf. [6-8], [13], [17], [18], [20-27])). A basic problem is the right

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