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CONTINUOUS DEPENDENCE ON MODELING IN THE CAUCHY PROBLEM FOR NONLINEAR ELLIPTIC EQUATIONS

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Abstract. The Cauchy problem for various types of second-order nonlinear elliptic equations is considered. A substitution $v = \varepsilon u$ in the equation leads to a perturbed equation whose solution is compared to an appropriate solution of an unperturbed second-order linear elliptic equation obtained by formally setting $\varepsilon = 0$. In each case, a logarithmic convexity argument is used to show that appropriately constrained solutions of the original equation (assumed to exist) differ from a solution of the associated linear equation in a manner depending continuously on the parameter ε.

1. Introduction. A problem in ordinary or partial differential equations is said to be properly posed if it has a unique solution in the class under consideration and if this solution depends continuously on the data in some appropriate measure. Otherwise the problem is said to be improperly posed. An improperly posed problem can sometimes be stabilized by imposing a constraint on the set of solutions to be considered. The papers of John [9] and Pucci [16] in the 1950's were among the first to recognize the importance of improperly posed problems in applications.

Several ill-posed Cauchy problems for quasi-linear elliptic equations and systems were investigated in the 1960's and 1970's. Payne [14] showed that the Cauchy problem for a second-order linear elliptic equation can be stabilized by imposing an *a priori* bound on the L^2-norm of the solution. Extensions of the result in [14] to the Cauchy problem for equations of the type

$$Lu = f(x, u, \operatorname{grad} u), \tag{1.1}$$

for a uniformly elliptic second-order linear operator L, are found in [6, 18–20]. In each case a Holder continuous dependence result is obtained by restricting the L^2-norm of the solution.

The techniques in [14] have also been modified to derive continuous dependence results for perturbed equations. Adelson's work [1, 2] concerns the feasibility of approximating a solution of a Cauchy problem for a perturbed equation by a solution of a Cauchy problem for the corresponding unperturbed equation. This concept will be referred to as the question of *continuous dependence on modeling* for the

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