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JUMPING NONLINEARITIES FOR A NONLINEAR EQUATION WITH AN N-th ORDER DISCONJUGATE OPERATOR*

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1. Introduction. This paper deals with the problem of jumping nonlinearities for equations of the form

$$L_n y + f(y) = h(x), \qquad 0 < x < T \tag{1.1}$$

subjected to boundary conditions at 0 and T.

In (1.1) f is a C^1 function defined on $\mathbb{R}$, L_n is an n-th order linear differential disconjugate operator and $h \in C[0,T]$. Also, $L_i y$, called the quasiderivative of order i of y, is recursively defined based on the functions $\rho_i \in C^{n-i}[0,T]$ into which L_n, as a disconjugate operator can be decomposed, namely,

$$L_i y = \rho_i (L_{i-1} y)', \quad i = 1, 2, \ldots, n \tag{1.2}$$

with

$$L_0 y = \rho_0 y \tag{1.3}$$

where $\rho_i(x) > 0$ for $x \in [0,T]$, $i = 1, 2, \ldots, n$. In (1.2) and henceforth, $' = \frac{d}{dx}$ and $L \equiv L_n$.

Next, let r_0, r be fixed nonnegative integers such that $0 \le r_0 \le r \le n-1$ and define $I_r = \{0, 1, \ldots, n-1\} \backslash \{r\}$. We shall say that $y \in BC(r_0, r, T)$ if $y \in C^{n-1}[0,T]$ and y satisfies the boundary conditions

$$(L_i y)(0) = 0, \quad i \in I_r; \quad (L_{r_0} y)(T) = 0. \tag{B.C.}$$

It is well known (see [6]), that the eigenvalues of the linear problem

$$Ly + \lambda y = 0; \quad y \in BC(r_0, r, T) \tag{1.4}$$

are simple, form an increasing sequence $\{\lambda_i\}_{i=1}^{\infty} \subseteq \mathbb{R}^+$ such that $\lim_{i \to \infty} \lambda_i = +\infty$, and that ϕ_1, an eigenfunction corresponding to λ_1, does not vanish on $(0,T)$. Let $s \in \mathbb{R}$ and write h in (1.1) as

$$h(x) = s\phi_1(x) + h_1(x) \tag{1.5}$$

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