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POSITIVE SOLUTIONS OF A SEMILINEAR ELLIPTIC EQUATION AND ITS ASYMPTOTIC BEHAVIOUR

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Abstract. The existence and the asymptotic behaviour of a class of semilinear elliptic problems are discussed.

1. Introduction. In this paper, we establish the existence of positive solutions for semilinear elliptic problems

$$-\Delta u = \lambda f(u-a) \quad \text{in } \mathbb{R}^N, \quad u(x) \to 0 \text{ and } |\nabla u(x)| \to 0 \text{ as } |x| \to \infty, \tag{P}$$

where f is "superlinear" at infinity, λ and a are positive constants; moreover, we investigate the limiting behaviour of solutions of (P) as λ and a are taken as parameters.

Problem (P) is a free boundary problem and the set $A = \{x \in \mathbb{R}^N : u(x) > a\}$ is called "core" of the solution u. This kind of problem arises in plasma physics (see [3] and references therein).

Problem (P) is approximated by Dirichlet problems on bounded domains

$$-\Delta u = \lambda f(u-a) \quad \text{in } \Omega, \quad u|_{\partial\Omega} = 0. \tag{P$_\Omega$}$$

In Section 2, an existence result is obtained by critical point theory for (P_Ω), moreover, some estimates for the critical values are established. The existence result for (P) is proved in Section 3. Let us remark that the existence of positive solutions of (P) is in contrast with the result of [5] (see Remark 7 below) where $a = 0$. Furthermore, we will also show that, roughly, solutions u_a of (P) tend to 0 as $a \to 0^+$ and solutions u_λ of (P) tend to a function with an isolated singularity as $\lambda \to +\infty$. For other results concerning the limiting behaviour of solutions of free-boundary problems like (P_Ω), we refer, for example, to [2, 4].

2. Problems in bounded domains. Let $\Omega \subset \mathbb{R}^N$ $(N \geq 3)$ be a bounded domain. Consider the problem

$$-\Delta u = \lambda f(u-q) \quad \text{in } \Omega \tag{2.1a}$$

$$u = 0 \quad \text{on } \partial\Omega. \tag{2.1b}$$

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